Matrix Factorization

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Supplement to Modeling

Eigenvalues and Eigenvectors

- Given a $k \times k$ matrix A we call a vector $\vec{p}$ eigenvector of A if

$$A \cdot \vec{p} = \lambda \cdot \vec{p}$$

The value λ is called eigenvalue

- A matrix A is positive definite, if for all vectors $\vec{p}$

$$\vec{p}' \cdot A \cdot \vec{p} \geq 0$$

Eigenvalues and Eigenvectors

- For a positive definite symmetric matrix all eigenvalues are positive and the eigenvectors for different eigenvalues are orthogonal
- A matrix P is called orthonormal matrix if the columns of P are all orthogonal
- Note that for an orthonormal matrix P the inverse matrix is defined by the transposed matrix

$$P^{-1} = P', \quad \Rightarrow P \cdot C = P' \cdot P = I$$

Eigenvalues and Eigenvectors

- Matrix factorization: For a positive definite symmetric matrix A there exist an orthonormal matrix P and a diagonal matrix D such that

$$A = P \cdot D \cdot P'$$

  The matrix P is defined by the eigenvectors and the matrix D by the eigenvalues

- Geometric interpretation: A defines a new coordinate system by rotation and change in the units of the coordinates

Application in dimensionality reduction

- For analytical purposes data in BI applications are frequently represented as a case by variates matrix. The rows represent the vector of observed attributes of the individual cases and the columns represent the different attributes
- A measure of similarity of two attributes A_1 and A_2 is the correlation, which is always a value in the interval [-1 , 1]

Application in dimensionality reduction

- Extreme cases:

$$\rho(A_1, A_2) = \begin{cases} 1 & \text{positiv correlated} \\ 0 & \text{uncorrelated} \\ -1 & \text{negative correlated} \end{cases}$$

- Research question: can we represent high dimensional observations in a low dimensional space such that the similarities between observations and the attributes are well visualized?

Application in dimensionality reduction

- The solution is obtained by the method of principal components:
  - Determine the eigenvectors and the eigenvalues of the correlation matrix of the observations
  - Calculate the coordinates of the attributes in the new coordinate system (factor loadings)
  - Calculate the coordinates of the observations in the new coordinate system (factor scores)
  - Visualize the first two coordinates of the attributes and the observations

Application in dimensionality reduction

- Example: In the CRM use case there was a survey about the user satisfaction with the company
- The correlation matrix of the answers is given by

	Comp	Kind	Help	Quick	Eco	Perf
Comp	1.00	0.89	0.99	0.79	0.56	0.76
Kind	0.89	1.00	0.85	0.84	0.43	0.68
Help	0.99	0.85	1.00	0.72	0.58	0.73
Quick	0.79	0.84	0.72	1.00	0.46	0.62
Eco	0.56	0.43	0.58	0.46	1.00	0.59
Per	0.76	0.68	0.73	0.62	0.59	1.00

Strength	
(grey)	no
(green)	weak
(orange)	medium
(red)	strong

Application in dimensionality reduction

- Representation with principal components