Convergence of a discrete Markov Chains generated from a contingency table

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```
library(expm)
```

```
## Warning: package 'expm' was built under R version 3.4.3
```

```
## Loading required package: Matrix
```

```
##
## Attaching package: 'expm'
```

```
## The following object is masked from 'package:Matrix':
##
##     expm
```

```
p1<-0.11
p2<-0.09
p3<-0.1
p4<-0.7

#Joint distribution of the variables X and Y

(f_xy<-matrix(c(p1,p2,p3,p4), nrow=2))
```

```
##      [,1] [,2]
## [1,] 0.11  0.1
## [2,] 0.09  0.7
```

```
# Marginal distribution of X
 (px_1<- p1+p3)
```

```
## [1] 0.21
```

```
 (px_2<- p2+p4)
```

```
## [1] 0.79
```

```
# Marginal distribution of y
 (py_1<- p1+p2)
```

```
## [1] 0.2
```

```
 (py_2<- p3+p4)
```

```
## [1] 0.8
```

```r
# Conditional distribution of Y given X
(Ay_x<- matrix(c(p1/(p1+p3), p3/(p1+p3),p2/(p2+p4),p4/(p2+p4)),nrow=2))
```

```
##           [,1]      [,2]
## [1,] 0.5238095 0.1139241
## [2,] 0.4761905 0.8860759
```

```r
# Conditional distribution of X given Y
(Ax_y<- matrix(c(p1/(p1+p2), p2/(p1+p2),p3/(p3+p4),p4/(p3+p4)),nrow=2))
```

```
##      [,1]  [,2]
## [1,] 0.55 0.125
## [2,] 0.45 0.875
```

```r
#Two-step transition probability for X in two steps
(Ax_x<-Ax_y%*%Ay_x)
```

```
##          [,1]      [,2]
## [1,] 0.347619 0.1734177
## [2,] 0.652381 0.8265823
```

```r
# Two-step transition probability for Y in two steps
(Ay_y<-Ay_x%*%Ax_y)
```

```
##           [,1]      [,2]
## [1,] 0.3393611 0.1651597
## [2,] 0.6606389 0.8348403
```

```r
#Convergence of the transition probability to the marginal probability

#Variable X

for(i in 1:20) {
     mat<-Ax_x%^%i
       cat("Iteration Nr:", i,"\n")
     print(mat) }
```

```
## Iteration Nr: 1
##          [,1]      [,2]
## [1,] 0.347619 0.1734177
## [2,] 0.652381 0.8265823
## Iteration Nr: 2
##           [,1]      [,2]
## [1,] 0.2339734 0.2036273
## [2,] 0.7660266 0.7963727
## Iteration Nr: 3
##           [,1]      [,2]
## [1,] 0.2141762 0.2088899
## [2,] 0.7858238 0.7911101
## Iteration Nr: 4
##           [,1]      [,2]
## [1,] 0.2107275 0.2098066
```

```
## [2,] 0.7892725 0.7901934
## Iteration Nr: 5
##           [,1]      [,2]
## [1,] 0.2101267 0.2099663
## [2,] 0.7898733 0.7900337
## Iteration Nr: 6
##           [,1]      [,2]
## [1,] 0.2100221 0.2099941
## [2,] 0.7899779 0.7900059
## Iteration Nr: 7
##           [,1]     [,2]
## [1,] 0.2100038 0.209999
## [2,] 0.7899962 0.790001
## Iteration Nr: 8
##           [,1]      [,2]
## [1,] 0.2100007 0.2099998
## [2,] 0.7899993 0.7900002
## Iteration Nr: 9
##           [,1] [,2]
## [1,] 0.2100001 0.21
## [2,] 0.7899999 0.79
## Iteration Nr: 10
##      [,1] [,2]
## [1,] 0.21 0.21
## [2,] 0.79 0.79
## Iteration Nr: 11
##      [,1] [,2]
## [1,] 0.21 0.21
## [2,] 0.79 0.79
## Iteration Nr: 12
##      [,1] [,2]
## [1,] 0.21 0.21
## [2,] 0.79 0.79
## Iteration Nr: 13
##      [,1] [,2]
## [1,] 0.21 0.21
## [2,] 0.79 0.79
## Iteration Nr: 14
##      [,1] [,2]
## [1,] 0.21 0.21
## [2,] 0.79 0.79
## Iteration Nr: 15
##      [,1] [,2]
## [1,] 0.21 0.21
## [2,] 0.79 0.79
## Iteration Nr: 16
##      [,1] [,2]
## [1,] 0.21 0.21
## [2,] 0.79 0.79
## Iteration Nr: 17
##      [,1] [,2]
```

```
## [1,] 0.21 0.21
## [2,] 0.79 0.79
## Iteration Nr: 18
##      [,1] [,2]
## [1,] 0.21 0.21
## [2,] 0.79 0.79
## Iteration Nr: 19
##      [,1] [,2]
## [1,] 0.21 0.21
## [2,] 0.79 0.79
## Iteration Nr: 20
##      [,1] [,2]
## [1,] 0.21 0.21
## [2,] 0.79 0.79
```

```
 #Variable Y

 (Ay_20<-Ay_y%^%20)
```

```
##      [,1] [,2]
## [1,]  0.2  0.2
## [2,]  0.8  0.8
```