Aufgaben zum Tutorium 2 - MG2

Unbestimmte Integrale

1) Bestimme die folgenden Integrale

- $\int x^3 + 2x^2 + 10 \, dx$

  $= \int x^3 dx + \int 2x^2 dx + \int 10 \, dx = \int x^3 dx + 2\int x^2 dx + 10 \int 1 \, dx$

  $= \frac{1}{4}x^4 + \frac{2}{3}x^3 + 10x + C$

- $\int \sin(x) - e^x \, dx$

  $= \int \sin(x) dx - \int e^x dx = -\cos(x) - e^x + C$

- $\int 5 e^{2x} dx$ $\qquad (e^{2x})' = 2e^{2x}$

  $= 5 \int e^{2x} dx = 5 \cdot \frac{1}{2} e^{2x} = 2{,}5 \, e^{2x}$ $\qquad \Leftrightarrow \underbrace{\frac{1}{2}(e^{2x})'}_{= (\frac{1}{2}e^{2x})'} = e^{2x}$

2) Bestimme die folgenden Integrale mittels partieller Integration

- $\int x^2 e^x dx$ $\qquad u = x^2, \, v' = e^x \Rightarrow u' = 2x, \, v = e^x$

  $= x^2 e^x - \int e^x 2x \, dx = x^2 \cdot e^x - 2 \int e^x x \, dx$ $\qquad u = x, \, v' = e^x \Rightarrow u' = 1, \, v = e^x$

  $= x^2 e^x - 2[x \cdot e^x - \int e^x dx] = x^2 e^x - 2[x e^x - e^x] + c = e^x \cdot (x^2 - 2x + 2) + c$

- $\int x \cos(x) dx$ $\qquad u = x, \, v' = \cos(x) \Rightarrow u' = 1, \, v = \sin(x)$

  $= x \cdot \sin(x) - \int 1 \cdot \sin(x) dx = x \sin(x) - \int \sin(x) dx = x \sin(x) + \cos(x) + c$

- $\int \frac{5+x}{x^2} dx$ $\qquad u = 5 + x, \, v' = \frac{1}{x^2} \Rightarrow u' = 1, \, v = \frac{-1}{x}$

  $= \frac{-(5+x)}{x} - \int \frac{-1}{x} dx = \frac{-(5+x)}{x} + \int \frac{1}{x} dx = \frac{-(5+x)}{x} + \log(x) + c$

  $= \frac{-5}{x} + \log(x) + 1 + c$

  $= \frac{-5}{x} + \log(x) + c_2$

- $\int e^x \cos(x)\, dx \qquad u=\cos(x),\ v'=e^x \Rightarrow v=e^x,\ u'=-\sin(x)$

$= e^x\cos(x) - \int e^x(-\sin(x))dx = e^x\cos(x) + \int e^x \sin(x)dx \qquad u=\sin(x),\ v'=e^x \Rightarrow u'=\cos(x),\ v=e^x$

$= e^x\cos(x) + \sin(x)e^x - \int e^x\cos(x)dx$

$\Leftrightarrow\ 2\int e^x\cos(x)dx = e^x(\cos(x)+\sin(x))$

$\Leftrightarrow\ \int e^x\cos(x)dx = \frac{1}{2}e^x(\cos(x)+\sin(x)) + c$

3) Bestimme die folgenden Integrale mittels Substitution

- $\int \cos(x)\, e^{\sin(x)} dx \qquad y=\sin(x) \Rightarrow dy=\cos(x)dx$

$= \int e^y \cos(x)dx = \int e^y dy = e^y + c = e^{\sin(x)} + c$

- $\int x\cdot\sqrt{15-x^2}\, dx \qquad y=15-x^2 \Rightarrow dy=-2x\,dx \Leftrightarrow \frac{1}{-2}dy = x\,dx$

$= \int \sqrt{y}\, x\,dx = \int \sqrt{y}\,\frac{1}{-2}dy = \frac{-1}{2}\int\sqrt{y}\,dy = \frac{-1}{2}\,\frac{2}{3}\,y^{3/2} + c = \frac{-1}{3}(15-x^2)^{3/2} + c$

- $\int 6x^2 \sin(x^3)\, dx \qquad y=x^3 \Rightarrow dy=3x^2dx$

$= 2\int \sin(y)\, 3x^2 dx = 2\int \sin(y)dy = 2(-\cos(y)) + c = -2\cos(x^3) + c$

- $\int \frac{1}{\sqrt{1-x^2}}dx \qquad x=\sin(y) \Rightarrow dx=\cos(y)dy$

$= \int \frac{1}{\sqrt{1-\sin^2(y)}}dx = \int \frac{1}{\sqrt{\cos^2(y)}}\cos(y)dy = \int 1\,dy = y + c = \arcsin(x) + c$