

Grenzwerte

6) Berechne die folgenden Grenzwerte:

$$\bullet \lim_{x \rightarrow 3} \frac{x-3}{x^3 - 3x^2 - 4x + 12}$$

$$= \lim_{x \rightarrow 3} \frac{x-3}{(x-3)(x^2-4)} = \lim_{x \rightarrow 3} \frac{1}{x^2-4} = \frac{1}{3^2-4} = \frac{1}{5}$$

$$\bullet \lim_{x \rightarrow 0} \frac{x^2}{\sin(x)}$$

$$\stackrel{\text{L'Hos-}}{\text{pital}} \lim_{x \rightarrow 0} \frac{2x}{\cos(x)} = \frac{2 \cdot 0}{\cos(0)} = \frac{0}{1} = 0$$

$$\bullet \lim_{x \rightarrow \infty} x^{\frac{1}{x}}$$

$$= \lim_{x \rightarrow \infty} e^{\frac{1}{x} \cdot \log(x)} \stackrel{\lim_{x \rightarrow \infty} e^{f(x)} = e^{\lim_{x \rightarrow \infty} f(x)}}{\downarrow} = e^{\lim_{x \rightarrow \infty} \frac{\log(x)}{x}} \stackrel{\text{L'Hos-}}{\text{pital}} = e^{\lim_{x \rightarrow \infty} \frac{1/x}{1}} = e^0 = 1$$

$$\bullet \lim_{x \rightarrow \infty} \frac{3^x}{3^{2x}}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{3^x} = 0$$

$$\bullet \lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x}$$

$$\stackrel{\text{L'Hos-}}{\text{pital}} \lim_{x \rightarrow 0} \frac{-\sin(x)}{1} = \frac{0}{1} = 0$$