This module is part of the

Memobust Handbook

on Methodology of Modern Business Statistics

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Method: Seasonal Adjustment of Economic Time Series

Contents

General section

1. Summary

Seasonal adjustment, which consists in the estimation and the removal of the seasonal variation from time series has a long tradition, documented in Zellner (1978), Hylleberg (1992) and, more recently, Bell, Holan and McElroy (2012). Since both seasonally adjusted series and seasonal component are unobserved components and, consequently, a given time series has an unknown composition, many methods and procedures have been proposed and implemented to perform the seasonal adjustment. In addition to the ARIMA (AutoRegressive Integrated Moving Average)-model based method, implemented in TRAMO-SEATS (Maravall, 2012) and to the moving average based method implemented in X-12-ARIMA (U.S. Census Bureau, 2012), seasonal adjustment may be performed by some more or less conventional methods, such as Structural Time Series (STS) models, Bayesian seasonal adjustment, signal-extraction methods, different non-parametric (like spline-based) methods etc. Although the two main-stream procedures, TRAMO-SEATS and X-12-ARIMA, are generally recognised and accepted as the leading procedures in a process of production of seasonally adjusted data in official statistics, it is still important to study the alternatives in order to encourage diversity in development of seasonal adjustment.

An outline of the several seasonal adjustment procedures used at the National Statistical Offices of the European Union is given in Fischer (1995). Although this document might look outdated it still contains interesting comparisons among several methods used in different national institutes in Europe. This document emphasises advantages of TRAMO-SEATS and X-12-ARIMA over the comparing methods DAINTIES, BV4, SABL, X-11 UK version and X-11-ARIMA. Note that some of the methods described in the document are no longer in use.

Since the time of publication of the mentioned document several new methods for seasonal adjustment have been proposed in the available literature. These methods arise because of the need to deal with some issues that ARIMA-model based methodologies have difficulty tackling. Real time signal extraction is one such methodology based on the Direct Filter Approach (Wildy, 2008), implemented in the R-package signal extraction (R Development Core Team, 2012) created by the same author. The author claims that this method has certain advantages over the ARIMA-model based methods with respect to the turning-point detection and other relevant timing issues.

Non-parametric methods such as STL allegedly generate robust estimates of the time series components not distorted by aberrant observations (outliers). See Cleveland (1990) and R-package STL for more details (R Development Core Team, 2012). Although robust to outliers, the STL-method has some disadvantages in official statistics. This procedure does not have full functionality needed to produce seasonally adjusted estimates in a way relevant to a government statistical agency. Furthermore, the development of this method seems to be stagnated during recent years.

Bayesian seasonal adjustment, originally proposed by Akaike (1980), has been developed and implemented in several software-platforms, such as R-package TIMSAC and SAS procedure TSBAYSEA (SAS Institute, 2009). However, such a methodology has not yet attracted attention of the national statistical institutes, due to its complexity and the required theoretical background necessary to deal with the Bayesian framework.

One of the alternative modelling frameworks, the STS-models, is recommended as a substitute to the two main methods in the ESS (European Statistical System) guidelines on seasonal adjustment (Eurostat, 2009), if certain conditions are satisfied. The use of some other alternative methods falls under the category "to be avoided".

The ESS guidelines on seasonal adjustment aim to achieve harmonisation of the member state's national practices by promoting the idea of best practices in seasonal adjustment. Although the guidelines work towards a unified framework for seasonal adjustment within the ESS, they are not supposed to put limitations on the use of other methods. Under appropriate circumstances some less conventional models might offer innovative solutions to certain re-occurring problems that the national statistical institutes (NSI) have to deal with in their daily work with seasonal adjustment.

The main focus of this module is put on description of the decomposition based on ARIMA models, on moving averages and on STS-models, while the other classes of models are not treated. Section 2 is organised as follows. Sections 2.1 and 2.2 describe the two main stages of the seasonal adjustment of a given time series through the most widespread procedures, i.e., TRAMO-SEATS and X-12-ARIMA (X-13-ARIMA-SEATS): the pre-treatment and the decomposition. In particular, section 2.1 deals with the pre-treatment of time series required by both procedures before the decomposition and section 2.2 gives an overview of the decomposition based on moving averages (or *ad hoc* filters) and ARIMA models. Section 2.3 presents the STS model based approach, highlighting features that make it an appealing tool for seasonal adjustment. Finally, referring to TRAMO-SEATS and X-12-ARIMA, section 2.4 details the seasonal adjustment process of time series, distinguishing and describing eight steps.

2. General description of the method

2.1 Pre-treatment

The most widespread procedures of seasonal adjustment, TRAMO-SEATS and X-12-ARIMA, require the pre-treatment of time series aimed at adjusting the original series for special effects before the decomposition. Usually these effects refer to calendar effects, outliers, particular events known a-priori and so forth and the adjustment is carried out through reg-ARIMA models. These are presented in this section, while a different approach is considered in section 2.3.

2.1.1 Reg-ARIMA models

ARIMA models, as discussed by Box and Jenkins (1976), represent a practical way of dealing with moving features of seasonal time series. A general multiplicative seasonal ARIMA model for a time series Y_t can be written

$$\phi(B)\ \Phi(B^s)\ (1-B)^d(1-B^s)^D Y_t = \theta(B)\ \Theta(B^s)\ a_t \tag{1}$$

where

- Y_t may be replaced by deviations from its mean, $Y_t - \mu$;
- B is the backshift operator, such that $BY_t = Y_{t-1}$;
- s is the seasonal period ($s = 12$ for monthly data, $s = 4$ for quarterly data, …);

- $\phi(B) = (1 - \phi_1 B - ... - \phi_p B^p)$ is the non-seasonal AutoRegressive (AR) polynomial of order p and $\Phi(B^s) = (1 - \Phi_1 B^s - ... - \Phi_P B^{sP})$ is the seasonal AR polynomial of order P;
- $(1 - B)^d$ and $(1 - B^s)^D$ imply, respectively, the non-seasonal differencing of order d and the seasonal differencing of order D (generally $d = 0, 1, 2$ and $D = 0, 1$);
- $\theta(B) = (1 - \theta_1 B - \ldots - \theta q B^q)$ is the non-seasonal moving average (MA) polynomial, $\Theta(B^s) = (1 - \Theta_1 B^s - ... - \Theta_Q B^{sQ})$ is the seasonal MA polynomial;
- $a_t \sim WN(0,\sigma^2)$ is a white noise process with mean zero and variance σ^2.

In order to build ARIMA models the so called Box-Jenkins approach is used. It consists of an iterative scheme containing three stages: *i)* model identification, i.e., the selection of a tentative model, in particular the selection of the degree of regular/seasonal differencing and the orders of the stationary AR and invertible MA polynomials; *ii)* estimation of $(p + P + q + Q)$ parameters of the AR and MA polynomials and of the white noise variance; *iii)* diagnostic checking, mainly based on model residuals, assumed to be normally, identically and independently distributed (n.i.i.d.), on statistical significance of parameters and on in-sample and out-of-sample forecast performance (useful references are Box and Jenkins (1976), Harvey (1989), Hendry (1995)).

In model identification, it is important to employ the smallest possible number of parameters for an adequate representation of the time series. This principle of parsimony is particularly important in time series analysis, because variables in a time series model are usually autocorrelated and cross correlated. If a model is not reasonably parsimonious, such correlations may lead to spurious relationships in the model.

Since this model building process is often complex and time consuming, the choice of the ARIMA model is often based either on information criteria[1] such as AIC (Akaike Information Criterion), its corrected version AICC, BIC (Bayesian Information Criterion) and others or on automatic procedures. As far as information criteria are concerned, they are expressed in terms of the maximum of log likelihood and a penalty function depending on the number of parameters. The use of these information criteria implies that if different models produce similar maximum values of the log likelihood, the model with fewer parameters should be preferred. On the contrary, an additional parameter should be added in the model only when the maximum value of the log likelihood increases substantially. Although the choice of the best information criterion is not an easy task, all information criteria share the general principle of parsimony.

With reference to the automatic procedure for ARIMA model identification, it is worth emphasising that model identification is the most important step in the model building process influencing parameters estimates, forecasting and decomposition. The availability of a powerful automatic procedure for model identification in the most widespread procedures used for seasonal adjustment (Gomez and Maravall, 2001) TRAMO-SEATS and X-12-ARIMA (X-13-ARIMA-SEATS), has greatly simplified the seasonal adjustment, allowing a massive treatment and decomposition of many seasonal time series and enhancing the overall quality of data.

[1] Useful suggestions for a proper use of the information criteria to compare several (reg-)ARIMA models can be found on the X12 user manual (Census Bureau, 2013).

A particular class of ARIMA models is the *airline model*, so called because applied to a series of airline passengers in Box and Jenkins (1976):

$$(1-B)^d(1-B^s)^D Y_t = (1-\theta B)(1-\Theta B^s)\, a_t\,. \quad (2)$$

It is a parsimonious model providing a good fit for many seasonal macroeconomic time series. Its parameters can be given a structural interpretation (see Kaiser and Maravall, 2001):

a) the trend behaviour becomes more and more stable when $\theta \rightarrow 1$;

b) the seasonal behaviour becomes more and more stable when $\Theta \rightarrow 1$.

Anyway, attention should be paid when, estimating an airline model, its parameter estimates are near the non-invertibility region (e.g., estimates of θ and/or Θ are $-.99$). In fact, two reasons can explain this result: either trend/seasonality are practically deterministic or the model is overdifferenced. Testing for the significance of a linear trend or seasonal dummies determines the correct explanation.

Before considering a time series appropriate for ARIMA models, several prior treatments (adjustments) are generally needed in order to:

- remove special effects such as working/trading day, Easter effects and other national holidays (*calendar effects*);
- correct outliers;
- deal with special events known a-priori through intervention variables.

These pre-adjustments are implemented using a regression ARIMA model (hereinafter reg-ARIMA model), also called time series regression model or dynamic regression model (Pankratz, 1991).

A reg-ARIMA model can be written as

$$Z_t = \Sigma\, \beta_i\, X_{i,t} + Y_t \quad (3)$$

where Z_t is the (observed) time series, the $X_{i,t}$ are regression variables observed concurrently with Z_t, the β_i are regression parameters and $Y_t = Z_t - \Sigma\, \beta_i\, X_{i,t}$, the time series of regression errors (hereinafter called *linearised* series), is assumed to follow the ARIMA model in (1). The expressions (1) and (3) define the general reg-ARIMA.

In the reg-ARIMA model written in (3), the regression variables $X_{i,t}$ affect the dependent series Z_t only at concurrent time points, i.e., model (3) does not explicitly consider lagged regression effects $X_{i,t-1}$. Moreover, regression variables are deterministic variables, whose future values can be exactly predicted with a null forecast error. Lagged and stochastic effects can be included in the reg-ARIMA models implemented in the most recent releases of TRAMO-SEATS.

In order to include regression variables in the model, user knowledge about the time series being treated is required. Some variables that are frequently used are generated by the programs used for the seasonal adjustment, while other specific variables needed to deal with specific effects/abrupts in time series can be create by the user. Next section deals with three main groups of regression variables: calendar variables, outliers and intervention variables.

2.1.2 *Regression variables*

A. Regression variables for calendar effects

Many economic time series, such as production, sales and turnover, are an aggregation of unobserved daily values and are compiled each month. These time series may contain two kinds of calendar effects: the *trading day effect* (or day-of-week effect) and moving holidays (e.g., Easter) that are set according to a lunar calendar.

The trading day effect results from a combination of an underlying weekly periodicity in the unobserved daily data along with how many times each day of the week occurs in a given month. For example, July 2013 began on a Monday, so there are five Mondays, Tuesdays and Wednesdays and four of each of the other days. In July 2011, there are five Fridays, Saturdays and Sundays and four of each of the other days. Thus, the weekly periodicity along with the different numbers of each weekday may considerably affect time series. This can be shown comparing the sample autocorrelation function of unadjusted data with the one of data adequately treated for trading day effects. In fact, when the time series being analysed is significantly affected by these effects, its sample autocorrelation function may be seriously distorted. Moreover, since the ARIMA model suggested by its profile is not a parsimonious and easily interpretable model, these effects must be properly accounted for before a meaningful analysis of the data can be conducted.

Methods used to deal with trading day effects are based on the counting of the number of specific weekdays in a given month t (i.e., the number of Mondays $W_{1,t}$, the number of Tuesdays $W_{2,t}$, …, the number of Sundays $W_{7,t}$). These counts are then used as regression variables and the total trading day effects can be written as

$$td(\xi_1, \ldots, \xi_7, W_{1,t}, \ldots, W_{7,t}) = \Sigma_{i=1,7}\, \xi_i\, W_{i,t} \qquad (4)$$

with ξ_i , i= 1, …, 7 representing the effects due to Mondays, …, Sundays (here ξ_i and $W_{i,t}$ play the same role as β_i and $X_{i,t}$ in equation (3)). To avoid multicollinearity and also to consider the non-seasonal part of the trading day effects (as required in seasonal adjustment), the trading day effects are constrained to vary around zero, i.e., their long run average is required to be null

$$1/n\ \Sigma_{t=1,n}\, \Sigma_{i=1,7}\, \xi_i\, W_{i,t} = 1/n\ \Sigma_{i=1,7}\, \xi_i\, \Sigma_{t=1,n}\, W_{i,t} = 0 \qquad (5)$$

where n = 12×28 because the calendar is periodic of 28 years (if only years not multiple of 400 are considered in the 28 year span). It follows that relation (5) is fulfilled for $\Sigma_{i=1,7}\, \xi_i = 0$, yielding $\xi_7 = -\, \Sigma_{i=1,6}\, \xi_i$, and therefore

$$td(\xi_1, \ldots, \xi_7, W_{1,t}, \ldots, W_{7,t}) = \Sigma_{i=1,6}\, \xi_i\, W_{i,t} - \Sigma_{i=1,6}\, \xi_i\, W_{7,t} = \Sigma_{i=1,6}\, \xi_i\, (W_{i,t} - W_{7,t})$$

$$TD(\xi_1, \ldots, \xi_6, D_{1,t}, \ldots, D_{6,t}) = \Sigma_{i=1,6}\, \xi_i\, D_{i,t} \qquad (6)$$

with $D_{i,t}$ representing the *contrast* variables built using the variable for Sunday, $W_{7,t}$. The use of Sunday in (6) to build contrast variables is usual in the literature. However, in a more general approach each day of the week could be used, depending on the features of the economic activities/domains being considered (see Attal-Toubert and Ladiray, 2011).

Additionally, another regression variable can be included to model the length of the months, namely the leap year variable $LY_t = \Sigma_{i=1,7}\, W_{i,t} - l_m$, where

$$l_m = \Sigma_{i=1,7}\, W_{i,s} \quad \text{for } m = \text{January, March, \ldots, December} \tag{7}$$

$$l_m = 1/n\, \Sigma_{t=1,n}\, \Sigma_{i=1,7}\, W_{i,s} = 28.25 \text{ for } s = \text{February and } n = 4,$$

is the average length of months. In particular, LY_t is not null only for the months of February (0.25 when the month t is a February with 28 days and – 0.75 when the month t is a February with 29 days).

Sometimes in the reg-ARIMA estimation stage, some trading day parameters may not be statistically significant. In these cases, it is important not to eliminate the insignificant parameters, because the whole set of variables has to be completely retained or completely removed. On the contrary, the effect due to leap year, when statistically insignificant, may be omitted.

There is a more parsimonious representation of the effects due to the composition of calendar based on one regression variable. It is supposed that Monday to Friday have similar effects, while Saturday is treated as contrast variable along with Sunday. Its final representation is:

$$WD(\xi, , D_t) = \xi\, D_t = \xi\, (\Sigma_{i=1,5}\, W_{i,t} - 5/2 \times \Sigma_{i=6,7}\, W_{i,t}). \tag{8}$$

Usually (6) and (8) are referred to as *trading day* effects and *working day* effects, respectively.

As far as the calendar adjustment for working/trading days is concerned, two aspects deserve to be stressed: the one refers to the treatment of the national (civil or religious) holidays falling on working/trading days (point of view of data producers); the other concerns the interpretation of working-day adjusted data when they are disseminated to users (point of view of data users).

1. Among the several methods existing to adjust for trading-day and holiday effects in monthly economic time series, two methodologies are widespread among NSIs (Roberts *et al.*, 2009): one based on the U.S. Census Bureau's X-12-ARIMA method and one developed by Eurostat and suggested in the ESS guidelines on seasonal adjustment (Eurostat, 2009).

a. According to the U.S. Census Bureau's X-12-ARIMA method, fixed national holidays falling on a particular date or on a particular working/trading day of a given month are expected to have fixed effects (not affecting other months) and, consequently, to be absorbed by the seasonal component of the series. There is no need to include further regressors for these holidays in the reg-ARIMA model.

b. According to Eurostat's method, fixed national holidays falling on trading/working days are included in the above mentioned regressors and treated as Sunday. These regressors, corrected for fixed holidays and called country specific regressors, are expressed as:

$$\begin{gathered}(\#\,\mathrm{Mon}_t - \#\,\mathrm{hol}_{\mathrm{Mon},t}) - (\#\,\mathrm{Sun}_t + \#\,\mathrm{hol}_{\mathrm{Mon},t}) \\ \ldots \\ (\#\,\mathrm{Sat}_t - \#\,\mathrm{hol}_{\mathrm{Sat},t}) - (\#\,\mathrm{Sun}_t + \#\,\mathrm{hol}_{\mathrm{Sat},t}),\end{gathered} \tag{9}$$

where $\#\,\mathrm{hol}_{\mathrm{Mon},t}$ is the number of fixed holidays falling on Monday for the month t, or

$$(\#\,\mathrm{Mon}_t - \#\,\mathrm{hol}_{\mathrm{Mon},t}) - (\#\,\mathrm{Sun}_t + \#\,\mathrm{hol}_{\mathrm{Mon},t}) \tag{10}$$

where # hol_t is the number of fixed holidays falling on Monday, Tuesday, …, Friday for the month t. The main drawback of these country specific regressors is that they show a seasonal pattern. The first panel of figure 1 represents the autoregressive spectrum of an example of the regressor described in equation (4): spectral peaks are evident at both calendar frequencies (vertical pink lines) and seasonal frequencies (vertical dotted red lines). As stressed in the guidelines on seasonal adjustment, regression variables related to calendar effects have to remove only the non-seasonal part of these effects, since the seasonal part will be removed in the decomposition stage. Since the variables described in equations (3) and (4) show seasonality, the non-seasonal part of the day-of-week composition of the month/quarter can be estimated by the deviation of the number of working/trading days from their long-term monthly/quarterly average, i.e., removing monthly or quarterly averages (computed on a calendar whose length is a multiple of 28 years).

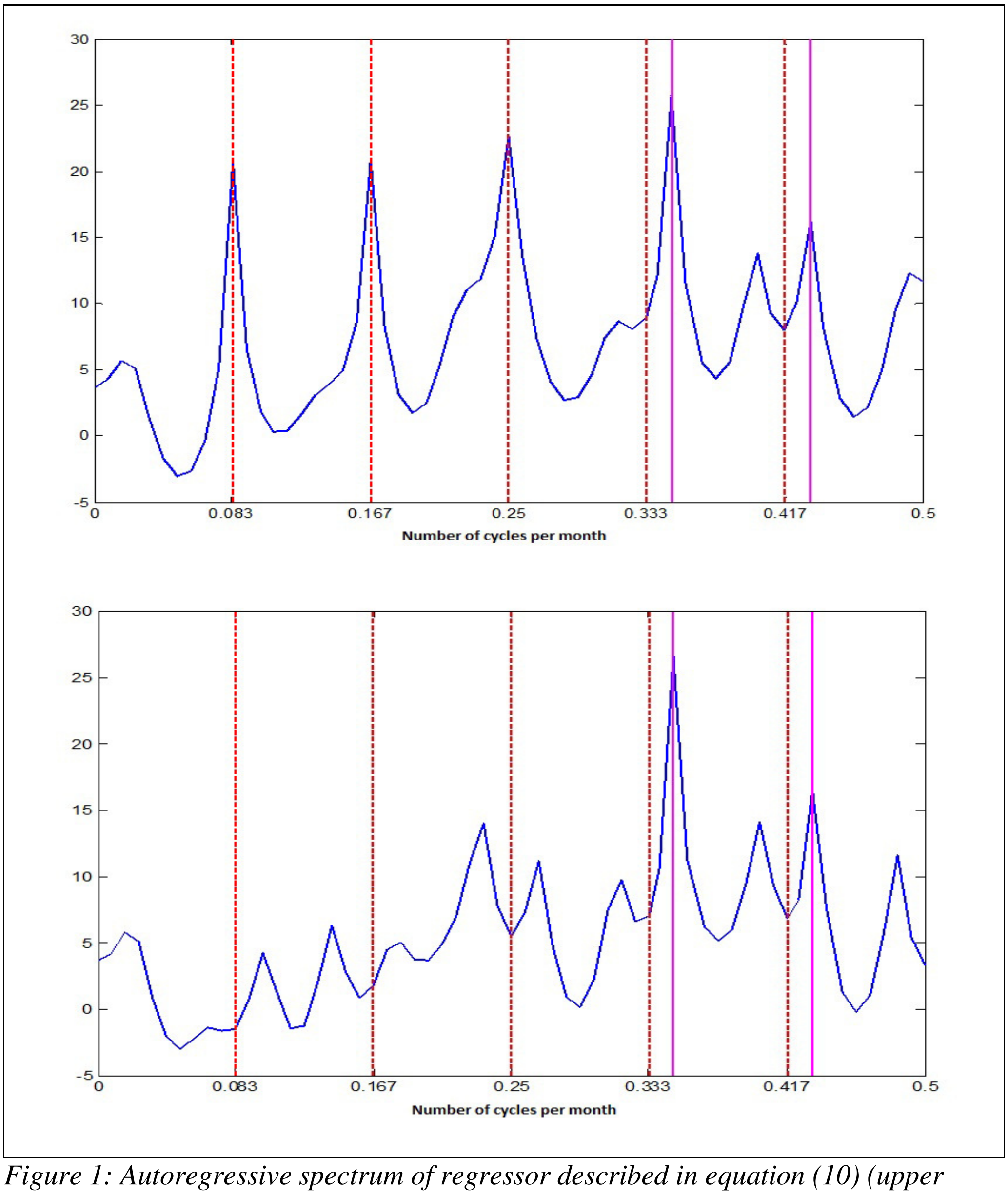

Figure 1: Autoregressive spectrum of regressor described in equation (10) (upper panel) and of its deseasonalised version (lower panel).

2. Another issue concerning the interpretation of calendar adjusted data (here the adjustment based on one regressor is considered) refers to comparison of y-o-y growth rates computed on both unadjusted and calendar adjusted data (in particular index number) when the same period, month or quarter, of the year y and y-1 have the same number of working days. In this case, in fact, their equality is expected. However, there may be cases where the equality is not fulfilled, in particular when the additive model is used. In fact, the additive adjustment for calendar adjustment is not proportional to the data level with the consequence that smallest data are overadjusted. Moreover the size of the difference between the two types of y-o-y growth rates (in case of additive model) depend on the size of the unadjusted y-o-y growth rates: the larger they are, the larger the difference is. This is shown in figure 2 where the difference between the y-o-y growth rates calculated on the unadjusted and the calendar adjusted data is reported on the vertical axis. It depends on the levels of data to be calendar adjusted (here the index numbers are considered) and on the size of the y-o-y growth rates of unadjusted data (in the figure they are displayed in percentages). For the multiplicative model, the light blue surface, intersecting the vertical axis at value zero, shows that when a period (month or quarter) has the same number of working days for two consecutive year (y and y-1) y-o-y growth rates on working day adjusted data are equal to y-o-y growth rates on unadjusted data (their difference is null as expected). On the contrary, for the additive model, small values (levels) are overadjusted and differences between unadjusted and working-day adjusted y-o-y growth rates are larger. This is emphasised when unadjusted low levels are associated with large (absolute) y-o-y growth rates. This situation is very common with time series featured by an important seasonal component with very small values in at least one period.

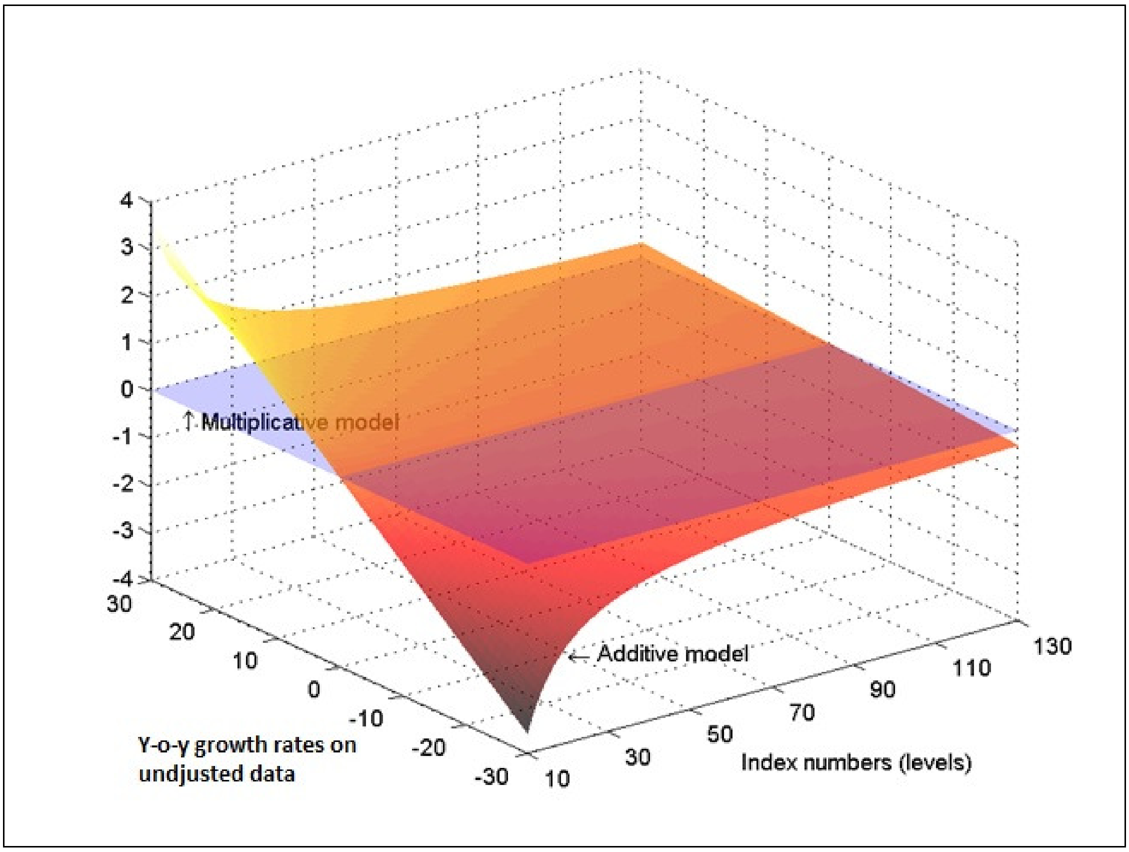

Figure 2: Differences between y-o-y growth rates computed on unadjusted and working day adjusted (reported on the vertical axis).

There are holidays that need a different correction because they are set according to the lunar calendar and, therefore, may fall in different days/months. One of them is Easter holiday. It represents a mobile holiday that may fall between the 22nd March and the 25th April, whose effects refers to the days/weeks before the holiday. An example is represented by the sales turnover that generally increases before Easter. For Christmas holiday, sales turnover also increases before the holiday, but it does not require a specific treatment because it falls in the same date every year.

Adjusting a time series for the Easter effects, therefore, requires a specific variable:

$$E_{\gamma,t}^{*} = (1/\gamma) \times n_t \tag{11}$$

where γ is the length (the number of days) of the Easter effect before Easter Sunday and n_t is the number of the γ days before Easter falling in month t. For example, if Easter falls the 4th April, under the hypothesis that the effects of the holiday lasts $\gamma = 6$ days, then this variable is null except in March and April, when it is 2/6 in March and 4/6 in April. In February it is nonzero only when $\gamma > 22$.

The deseasonalised and actual used version of this variable is obtained by removing the long run monthly averages of $E_{\gamma,t}^{*}$ computed on a long period (in X-12/X-13 a 500 year period of the Gregorian calendar is considered).

$$E_{\gamma,t} = E_{\gamma,t}^{*} - 1/T\ \Sigma_{t=1,T}\ E_{\gamma,t}^{*} \tag{12}$$

where T is the number of years in the period considered to calculate the averages.

The preceding paragraphs are based on three assumptions. Firstly, time series are available at monthly frequency. However, trading/working day effects can be found also in quarterly series, although they are not very common because the calendar composition of quarters does not change over time as that of months. In these cases, regressors are built counting the days of the week over the quarters. Secondly, time series are compiled aggregating daily values (flow series). If the series instead are compiled using the values at the end of the month (stock series), then different regression variables have to be used for an adequate adjustment of calendar effects (see Bell, 1984a and 1995, and Findley and Monsell, 2009). Thirdly, calendar effects are modelled through deterministic regression variables. In the ARIMA model based decomposition, Maravall and Pérez (2012) propose a stochastic trading/working day component (when the ARIMA model contains a regular AR polynomial, whose complex root has an associated frequency approximately equal to the theoretical trading/working day frequency)

B. Outliers

Macroeconomic time series are often subject to external events or abrupt changes such as introduction of new laws/regulations, sales promotions, strikes, recording errors and so forth. When these events are unexpected and their timing is unknown (e.g., recording errors), they are referred to as outliers, i.e., unusual observations that have a substantial impact on the time series and, consequently, on their analysis. Although several methods have been proposed for detection and adjustment of outliers, usually an automatic approach is used based on an iterative procedure (for details see Chen and Liu, 1993 and Gomez and Maravall, 2001a).

There are several reasons for outlier detection and adjustment in time series analysis (Pankratz, 1991):

a. understanding the time series under study;

b. discovering spurious observations such as recording errors;

c. simplifying the structure of the model and improving parameter estimates;

d. improving the forecasting performance.

All these motivations may have moderate to substantial impact on the seasonal adjustment of time series, in particular the improvement of the estimation of components (especially in an ARIMA model based approach) and the reduction of the revision size for seasonally adjusted data (when new observations are added).

In this section four types of outliers are presented, while their allocation to the different components is considered in section 3.

1. Additive outlier (AO)

   An additive outlier is an event that affects a time series for one period only, $t = t_0$. It can be represented through a *pulse* function:

$$P_t(t_0) = 1 \quad \text{for } t = t_0, \quad P_t(t_0) = 0 \quad \text{for } t \neq t_0.$$

   The reg-ARIMA model for the time series is

$$Z_t = \omega_{AO} P_t(t_0) + Y_t$$

where the value ω_{AO}, to be estimated, represents the deviation from the "true" value of Y_t and Y_t is assumed to follow the ARIMA model in (1).

2. Level shift (LS)

   A level shift is an event that affects a time series permanently from a period $t = t_0$ onward. It can be represented by a *step* function:

$$S_t(t_0) = -1 \quad \text{for } t < t_0, \quad S_t(t_0) = 0 \quad \text{for } t \geq t_0.$$

   The reg-ARIMA model for the time series is

$$Z_t = \omega_{LS} S_t(t_0) + Y_t$$

   where the term $\omega_{LS} S_t(t_0)$ adjusts for the level of the time series Z_t in first part, adapting it to the one of second part.

3. Temporary change (TC)

   A temporary change is an event that has an initial impact on the time series at $t = t_0$ and whose effect decays exponentially according to a factor $\delta \in (0,1)$, called dampening factor (i.e., the rate of decay back to the previous level of the time series):

$$T_t(t_0) = \delta^{t-t_0} \text{ for } t \geq t_0, \quad T_t(t_0) = 0 \quad \text{for } t < t_0.$$

   The reg-ARIMA model for the time series is

$$Z_t = \omega_{TC} T_t(t_0) + Y_t.$$

4. Seasonal outliers (SO)

   A seasonal outlier is an event that affects one period (month or quarter) of a time series permanently from time $t = t_0$ onward (Kaiser and Maravall, 2003). It can be represented by the following function (assuring null annual averages):

$$SO_t(t_0) = \begin{cases} 1 & \text{for } t < t_0 \text{ and } t \text{ same month/quarter as } t_0 \\ 0 & \text{for } t \geq t_0 \\ -(s-1)^{-1} & \text{otherwise.} \end{cases}$$

   where s is the seasonal period (s = 12 for monthly data, s = 4 for quarterly data).

   The reg-ARIMA model for the time series is

$$Z_t = \omega_{SO} SO_t(t_0) + Y_t$$

   where the term $\omega_{SO}\, SO_t(t_0)$ adjusts for the level of the month/quarter of the time series Z_t in first part and slightly modifies the level of the other months/quarters. As requirement of seasonal adjustment, the annual sums of the variable $SO_t(t_0)$ are always null. In fact, in the decomposition of a time series the SO are assigned to the seasonal component and, therefore, have to be removed from the seasonally adjusted series without modifying the annual sums (or averages) of the unadjusted series.

There is another type of outlier, called innovational outlier (IO), which affects a time series from a period $t = t_0$ onward according to the ARIMA model of the process. It can be considered an AO altering the white noise process a_t (see Chang, Tiao and Chen (1988) for further details and references). It is not considered here as it cannot be treated in the decomposition.

As far as outliers are concerned, the issue of detect an outlier at the end of a time series has to be stressed. In order to identify the type of an outlier some observations after the time of the occurrence of the event are needed. When the event occur at the end of the series under study, we are able to detect the outlier (unless its effects are moderate or negligible), but we cannot identify its nature (type). Although this inability affects neither the estimates of the model parameters, nor the estimated seasonally adjusted series (unless the detected outlier is a SO), it can seriously affect the estimation of the other components (i.e., trend and irregular) and the forecasting of both the unadjusted series and its components. As a consequence, attention should be paid when an outlier is detected at the end of the series. Some recommendations are listed below:

1. avoiding outliers at the end of the time series, unless they have a substantial impact on the parameter estimates;
2. if an outlier is detected at the end of the series, information should be collected to explain the reason of the outlier;
3. when an outlier at the end of the series is included in the model, its type should be checked as new observations become available.

As final remark, it is worth noting that all the outliers considered in this section can be detected automatically in the most recent releases of X-13 and TRAMO-SEATS. However, as far as the detection of SO is concerned, the plot of seasonal-irregular ratios computed on the preliminary components before adjusting for outliers may be very useful.

<u>C. Intervention variables</u>

As already said, macroeconomic time series are often subject to external events or abrupt changes such as introduction of new laws/regulations, sales promotions, strikes, recording errors and so forth. When these events are known (e.g., introduction of new laws/regulations) they are referred to as interventions. Intervention analysis is the method to incorporate such effects on the models. It is not considered in this section (an exhaustive treatment is presented in Box and Tiao, 1975).

2.2 Decomposition in TRAMO-SEATS and X-12-ARIMA

Completed the preliminary treatment aimed at removing the calendar effects, the outliers and other deterministic effects and estimating possible missing values, the resulting time series (the so-called *linearised series*[2]) are decomposed into the unobservable or latent components trend-cycle, seasonality and irregular. The most widespread procedures used by NSIs and other international agencies to produce official seasonally adjusted data are TRAMO-SEATS and X-12-ARIMA (they are also suggested by the ESS guidelines on seasonal adjustment). The former implement an ARIMA model-based decomposition, while the latter decompose a time series applying moving averages according to a recursive approach. Notwithstanding, these procedures have some common features: firstly, the models used are linear stochastic processes parametrised in the ARIMA-type format; secondly, to fulfil the previous assumption, the series needs some modification, called pre-treatment. So, assuming

[2] It is called linearised series because it can be assumed to be generated by a linear process.

an additive decomposition, the seasonal adjustment performed through the two approaches can be set in a unique framework described in figure 3. Given an observed time series, X_t, a reg-ARIMA model is estimated on it to derive: *i*) the regression effects, representing the deterministic part of the series; *ii*) the autocorrelated disturbance of the regression, modelled with an ARIMA model, representing the purely stochastic part of the series (i.e., the linearised series Y_t). This latter is decomposed, obtaining the stochastic components, hereinafter called simply components. The final components are derived summing up the regression effects to the components, according to their nature. Considering only the most frequent effects treated, the following rules are generally considered:

1) calendar effects and seasonal outliers are assigned to the seasonal component (so they do not appear in the seasonally adjusted series);
2) level shifts and ramp effects are assigned to the cycle trend;
3) transitory changes and additive outliers are assigned to the irregular component.

There is another practical reason to require pre-adjustment: filters used to estimate the components are two-sided filters involving past, present and future observations (and consequently past, present and future outliers or special effects/events), that is

$S_t = \ldots + \upsilon_{-2}\, Y_{t-2} + \upsilon_{-1}\, Y_{t-1} + \upsilon_0\, Y_t + \upsilon_1\, Y_{t+1} + \upsilon_2\, Y_{t+2} + \ldots$

$= (\ldots + \upsilon_{-2}\, B^2 + \upsilon_{-1}\, B + \upsilon_0 + \upsilon_1\, F + \upsilon_2\, F^2 + \ldots)\, Y_t = \upsilon(B, F)\, Y_t.$

In order to avoid this, such effects are removed and, after the decomposition, they are re-assigned to the components.

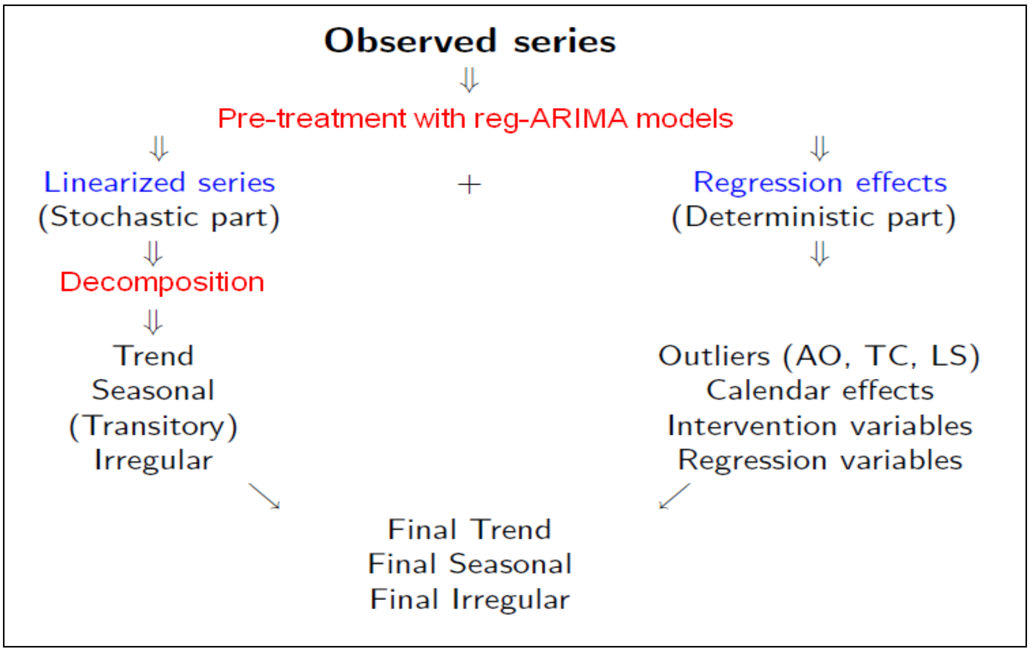

Figure 3: Pre-treatment and decomposition of a time series using reg-ARIMA models.

This section focuses on the decomposition of the linearised series (left side of figure 3): the approach based on moving averages (*ad hoc* filters) is presented in subsection 2.2.1, the ARIMA model based approach is described in subsection 2.2.2.

2.2.1 The moving averages based decomposition of X-12-ARIMA

The X-12-ARIMA decomposition can be viewed as sophisticated use of non-parametric smoothing based on filtering techniques. The two elements are combined into an algorithm that also takes into account extreme observations.

Moving averages

A time series can be smoothed by using the three-term simple (equal weights) moving average

$$P_t = (Y_{t-1} + Y_t + Y_{t+1})/3 \tag{13}$$

This method is called a 3x1 moving average. One may perform such smoothing twice to obtain a 3x3 moving average. That is, the smoothed series is calculated as a three-term simple moving average of a three-term simple moving average. Then, it follows that

$$P_t = (Y_{t-2} + 2Y_{t-1} + 3Y_t + 2Y_{t+1} + Y_{t+2})/9 \tag{14}$$

The centre of four successive observations is between two time periods. The mean of two four-term simple averages is, however, centred at a time period. This is called a 2x4 moving average and the expression is

$$P_t = (Y_{t-2} + 2Y_{t-1} + 2Y_t + 2Y_{t+1} + Y_{t+2})/8 \tag{15}$$

Equations (14) and (15) are examples of two five-term general moving averages. The set of weights is also called a filter and in this case the filter length is five. One may write these two filters in a more compact form as [1,2,3,2,1]/9 and [1,2,2,2,1]/8. Since we have symmetry this can also be written as [1,2,$\underline{3}$]/9 and [1,2,$\underline{2}$]/8 (centre underlined).

With a fixed filter length, the variance is minimal when the weights are equal. Of course, one can always reduce variance by increasing the filter length. The best filter reduces variance (eliminate noise) without losing too much relevant information. Accordingly, the filter length depends on the variability of the series.

At the beginning and the end of the series asymmetric filters can be used to solve the problem of non-available observations. An example of an asymmetric filter is [-0.034, 0.116, 0.383, 0.534, 0, 0, 0]. This filter is an asymmetric variant (Musgrave) of the 7-term Henderson filter. More details about Henderson filters can be found below.

The initial decomposition

Below we consider both the additive ($Y_t = T_t + S_t + I_t$) and the multiplicative model ($Y_t = T_t S_t I_t$).

For monthly data the initial estimate of the trend is found by using a 2x12 moving average. This 13-term filter is also known as a centred 12-term moving average. The weights are simply [1/2,1,1,1,1,1,1]/12. Thus

$$T_t = \left(\tfrac{1}{2}Y_{t-6} + Y_{t-5} + Y_{t-4} + \cdots + Y_t + \cdots + Y_{t+5} + \tfrac{1}{2}Y_{t+6}\right)/12 \tag{16}$$

With this filter length all months are equally weighted. Therefore, a stable seasonal component will not affect this trend estimate.

We now calculate the so-called SI-ratios, denoted as SI (not necessarily S times I). Note that, within this X-12 framework, SI is not necessarily S times I and neither it is an abbreviation of "seasonal index".

$$SI_t = Y_t - T_t \tag{17}$$

for additive models or

$$SI_t = Y_t / T_t \tag{18}$$

for multiplicative models. So in the case of a multiplicative model, the SI-ratio is $S_t * I_t$ and it is the ratio Y_t / T_t. Preliminary seasonal factors are calculated by using the 3x3 moving average to each month.

$$\widehat{S}_t = \left(SI_{t-24} + 2SI_{t-12} + 3SI_t + 2SI_{t+12} + SI_{t+24}\right)/9 \tag{19}$$

To normalise these, averages over 12-month periods are calculated. That is, 2x12 moving averages are calculated.

$$\widetilde{S}_t = \left(\tfrac{1}{2}\widehat{S}_{t-6} + \widehat{S}_{t-5} + \widehat{S}_{t-4} + \cdots + \widehat{S}_t + \cdots + \widehat{S}_{t+5} + \tfrac{1}{2}\widehat{S}_{t+6}\right)/12 \tag{20}$$

The seasonal components are now found as

$$S_t = \widehat{S}_t - \widetilde{S}_t \tag{21}$$

for additive models and for multiplicative models as

$$S_t = \widehat{S}_t / \widetilde{S}_t \tag{22}$$

The final decomposition is an improvement of this initial estimate. The underlying idea is based on two elements: How the trend can be estimated from a time series without seasonality and how the seasonal component can be estimated from a time series without a trend.

Finding a trend when seasonality is not present

To find the trend when seasonality is not present is a question of smoothing the time series. The initial trend estimate used equal weights for most months. Curvature trends are, however, better fitted using different weights. In fact, one may use negative weights at the ends. This is the case for the so-called Henderson filters (Henderson, 1916) which is used by X-12-ARIMA to obtain the trend. These filters are constructed so that filtering of third degree polynomials leave the time series unchanged. Another criterion is that the sequence of weights should be as smooth as possible. Further details can be found in Ladiray and Quenneville (2001). As noted in this reference, the coefficients of these moving averages may be calculated explicitly. For an order 2p+1 average, by letting n=p+2, the coefficients for $i=-p,\ldots,p$ can be written as

$$\frac{315[(n-1)^2-i^2][n^2-i^2][(n+1)^2-i^2][3n^2-16-11i^2]}{8n(n^2-1)(4n^2-1)(4n^2-9)(4n^2-25)} \tag{23}$$

When using X-12-ARIMA it is possible to specify the filter length manually. Otherwise, the default automatic method will, in the case of monthly series, use 9, 13 or 23 terms. For quarterly series, the program will choose either a 5- or a 7-term Henderson moving average. Using the notation above, the weights of these two filters are [-21,84,<u>160</u>]/286 and [-42,42,210,<u>295</u>]/715, respectively. The 13-term Henderson filter is [-325,-468,0,1100,2475,3600,<u>4032</u>]/16796.

Finding the seasonality when a trend is not present

The seasonal component is found by applying moving averages to each month or quarter. One alternative is to calculate the simple average of all the values for each month or quarter. This is called a stable seasonal filter. When using other filters it is allowed the seasonal component to vary along time. The older versions of X-12-ARIMA used a 3x5 moving average as default. This is a 7-term filter with weights [1,2,3,<u>3</u>]/15. For monthly data this means that a seasonal average would be calculated as

$$P_t = \left(Y_{t-36} + 2Y_{t-24} + 3Y_{t-12} + 3Y_t + 3Y_{t+12} + 2Y_{t+24} + Y_{t+36}\right)/15 \tag{24}$$

Other possible filters are, 3x1, 3x3, 3x9 and 3x15. Note that the latter filter is simply [1,2,3,3,3,3,3,3,<u>3</u>]/45. By default, the filter type is selected automatically by the program.

The combined algorithm

As mentioned above, asymmetric filters can be used at the beginning and the end of the series. However, X-12-ARIMA still uses symmetric Henderson filters to calculate the trend at the end of the series. Forecasts from the reg-ARIMA modelling are then used in place of the unobserved values.

The algorithm for calculating the trend and seasonal components starts with initial estimates as described above. The whole algorithm involves several steps. One element is downweighting of observations with an extreme irregular component. Iterations are therefore needed. At each stage it is possible to obtain a seasonal adjusted series (seasonality not present) or a series based only on the seasonal and irregular components (trend not present). This way the final estimates are calculated according to text above (*"Finding a trend when seasonality is not present"* and *"Finding the seasonality when a trend is not present"*). For details, see Dagum (1980), Findley et al. (1998) and Ladiray and Quenneville (2001).

2.2.2 The ARIMA model based (AMB) decomposition of SEATS

In the ARIMA model based decomposition, filters depend on the time series features because they are derived from the ARIMA model estimated on the data. Moreover, it is possible to do inference on the estimated components (because their theoretical properties are known) and to derive forecasts for the components together with their confidence intervals.

In order to understand how a filter can depend on the time series features let us consider the following examples (drawn from Maravall, 2012):

1) $Y_t = a_t$, with $a_t \sim$ WN$(0,\sigma_a^2)$

   The time series is not seasonal, that is the seasonal component $S_t = 0$, so the filter applied to Y_t to derive $s_t = 0$ should be $\upsilon(B, F)$=0.

2) $(1 + B + \ldots + B^s)Y_t = w_t$, with w_t having an MA structure

   The time series is the seasonal component, that is $S_t = Y_t$, so the filter applied to Y_t should be $\upsilon(B, F)$=1.

Figure 4 represents the steps of the AMB decomposition. Given the model for Y_t, firstly the models of the unobserved components are derived (if an acceptable decomposition exists), then the Minimum Mean Square Error (MMSE) estimators for components are computed and finally component estimates are derived.

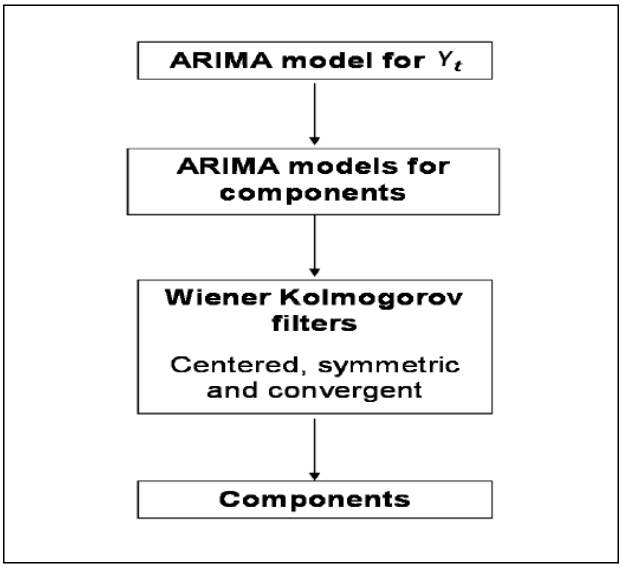

Figure 4: A representation of the AMB decomposition method.

I. Model for Y_t

Given an observed time series, the first step is the identification of the (multiplicative seasonal) ARIMA model:

$$\phi(B)\Phi(B^s)\nabla^d\nabla_s^D Y_t = \theta(B)\Theta(B^s)a_t, \quad a_t \sim WN(0,\sigma_a^2),$$

Using a more compact notation, it can be re-written as

$$\phi_Y(B)Y_t = \theta_Y(B)a_t.$$

It is worth stressing that the previous model is, in general, invertible and non-stationary. In particular non-stationarity, $d, D > 0$, allows for evolving trend and seasonal component whose features change over time.

II. Decomposition of the model for Y_t

The AR polynomial $\phi_Y(B)$ is factorised, allowing the definition of the components of a given series. For example, if the AR polynomial $\phi_Y(B)$ is $\nabla\nabla_4$ (the product of the regular and the seasonal differencing operators), then it can be factorised as $\nabla\nabla_4 = (1-B)(1+B^4) = (1-B)^2$ $(1+B+B^2+B^3) = \nabla^2 S$, where the factor ∇^2 implies the presence of the trend and the factor S (the annual aggregation operator) implies the presence of the seasonal component. Therefore, the series can be decomposed into trend, seasonality and irregular:

$$Y_t = T_t + S_t + I_t\,. \tag{25}$$

These components are assumed to follow ARIMA models

$$\phi_T(B)T_t = \theta_T(B)a_{T,t} \tag{26}$$

$$\phi_S(B)S_t = \theta_S(B)a_{S,t} \tag{27}$$

$$I_t = a_{I,t} \tag{28}$$

where $\phi_i(B)$ and $\theta_i(B)$, $i = P, S$, are finite polynomials in B of order p_i and q_i, respectively, having no common zeros and all zeros lying on or outside the unite circle.

As far as the ARIMA model for the trend is concerned (equation 24), generally $\phi_T(B)$ is non-stationary since it contains the regular differencing operator, either $\nabla = (1-B)$ or $\nabla^2 = (1-B)^2$, while the r.h.s. of the model allows the trend to evolve over time (i.e., a stochastic trend). The upper two panels of figure 5 compare a deterministic and a stochastic trend. With reference to the model for the seasonal component (equation 25), usually $\phi_S(B)$ is non-stationary and contains the annual aggregation operator, $S = (1+B+B^2+B^3)$ for quarterly series or $S = (1+B+B^2+...+B^{11})$ for monthly series; the r.h.s. of the model allows the seasonal component to evolve over time but preserving regular fluctuation locally. In the lower two panels of figure 5 a deterministic and a stochastic seasonal component are displayed.

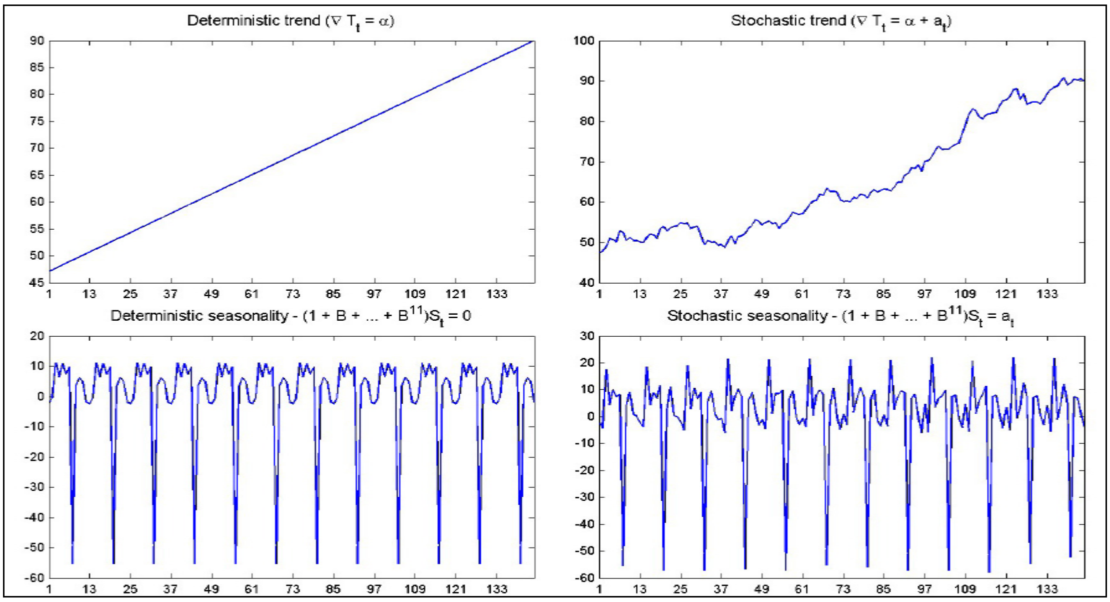

Figure 5: Deterministic and stochastic components.

In the representation (24-26) the following assumptions are fulfilled:

1) the variables $a_{T,t}$, $a_{S,t}$ and $a_{I,t}$ are mutually independent white noise processes, identically and independently distributed as $N(0,\sigma_T^2)$, $N(0,\sigma_S^2)$ and $N(0,\sigma_I^2)$;
2) the autoregressive polynomials $\phi_T(B)$ and $\phi_S(B)$ do not share common roots;
3) the moving average polynomials $\theta_T(B)$ and $\theta_S(B)$ have roots lying on and outside the unit circle and do not share unit common zeros.

The first assumption implies independent components and is based on the consideration that causes of the different components are not much related (e.g., weather causes seasonal fluctuations, while technology and investment cause the evolution of the trend); the second assumption implies that different components (generally non-stationary) are associated with different spectral peaks; the third assumption admits non invertible components and guarantees the invertibility of the model for Y_t.

Exploiting a different representation of the models for both Y_t and components

$$Y_t = \frac{\theta_Y(B)}{\phi_Y(B)} a_t, \; T_t = \frac{\theta_T(B)}{\phi_T(B)} a_{T,t} \text{ and } S_t = \frac{\theta_S(B)}{\phi_S(B)} a_{S,t}$$

from relation (25) the following identity can be derived

$$\frac{\theta_Y(B)}{\phi_Y(B)} a_t = \frac{\theta_T(B)}{\phi_T(B)} a_{T,t} + \frac{\theta_S(B)}{\phi_S(B)} a_{S,t} + a_{I,t}$$

Multiplying both sides for the factorisation of $\phi_Y(B)$, i.e., $\phi_T(B)\phi_S(B)$, the following identity is obtained

$$\theta_Y(B)a_t = \theta_T(B)\phi_S(B)a_{P,t} + \theta_S(B)\phi_T(B)a_{S,t} + \phi_T(B)\phi_S(B)a_{I,t}.$$

Assuming, in general, that $\theta_T(B)$ and $\theta_S(B)$ have the same order of $\phi_T(B)$ and $\phi_S(B)$, respectively, by equating the autocovariance function of both sides one can get a system of equations whose unknowns are the parameters of $\theta_T(B)$, $\theta_S(B)$ and the variances σ_T^2, σ_S^2 and σ_I^2. Two issues have to be stressed:

a) some models do not admit a decomposition, because some components may have a negative spectra;
b) if a model admits a decomposition, since the number of unknowns are greater than the number of equations, infinite decompositions exist and a choice must be made. This underidentification problem is solved through the *canonical decomposition*, i.e., the decomposition that maximises the variance σ_I^2 and, therefore, minimises the variances σ_T^2 and σ_S^2. Minimising the latter variances means that the trend and seasonal component are made as stable as possible, remaining compatible with the model for Y_t, and their models became noninvertible.

III. Estimators for the components

The optimal estimators of the trend, seasonal and irregular component are computed as the MMSE estimators, that is as a conditional expectation of S_t given $\{ Y_1, Y_2, ..., Y_T \}$ (here S_t represents the more generic signal)

$$\hat{S}_t = E(S_t \mid Y_1, Y_2, ..., Y_T)$$

Assuming the multivariate normal distribution, this conditional expectation is a linear combination of $Y_1, Y_2, ..., Y_T$ and it can be obtained through either the Kalman filter or the Wiener-Kolmogorov (WK) filter. The latter is considered because it is more useful for analysis.

a) *Historical or final estimators* ($T \to \infty$)

$$\hat{S}_t = \upsilon(B, F) Y_t = \left[\upsilon_0 + \sum_{j=1}^{\infty} \upsilon_j (B + F) \right] Y_t$$

where $\upsilon(B, F)$ represents the WK filter, that is shown to be centred in *t*, symmetric and convergent in B and F as it represents the autocovariance generating function of a stationary model.

b) *Preliminary estimators* (*finite realisation*)

$$\hat{S}_{t|T} = \upsilon^t (B, F) Y^e_{t|T}$$

where $\upsilon^t (B, F)$ is the truncated filter and $Y^e_{t|T}$ is the "extended" series, i.e., the series extended with forecasts and backcasts, with $Y^e_{t|T} = Y_t$ if $t <= T$ and $Y^e_{t|T}$ is the forecast or the backcast if $t > T$ or $t < 1$. In the particular case $t = T$, $\hat{S}_{T|T}$ is called concurrent estimator.

Figure 6 shows some examples of WK filters to derive the historical estimates of the components.

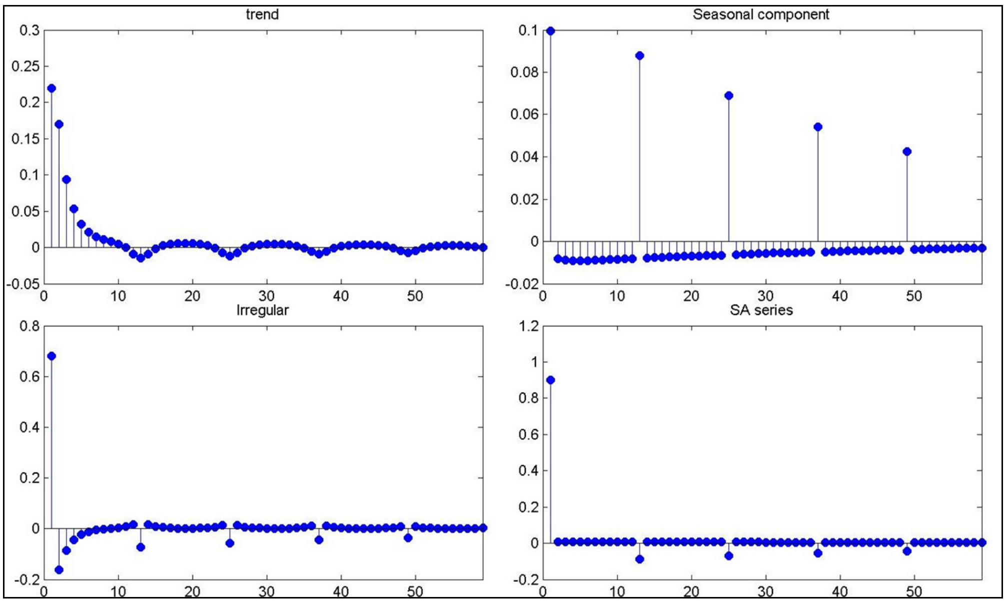

Figure 6: Examples of WK filters

IV. Computation of component estimates: some remarks

The application of the filters (derived from the ARIMA models) to the observed (linearised) series produces the estimates for the components. The comparison of their properties with the (theoretical) properties of estimators (available only in a model-based context) represents a useful diagnostic tool to assess the decomposition.

1) Convergence is an important property of filters since it allows us to truncate them when applied to the observed (linearised and extended) series. In many applications it is reached after three-five years, so with a time series of 20 years, the estimates for the central years (10-14 years) can be considered final.
2) Symmetry of filters requires the extension of the series with forecasts. As new observations become available, forecasts are replaced with new data and therefore previous component estimates are revised. The revision size depends on the forecast error: the better the series can be forecasted, the smaller the revisions in the preliminary estimates will be. This stresses the importance of the identification of the ARIMA model for the observed series, from which depend the properties of both the component and the estimators.
3) Generally, for stable components the convergence of the preliminary estimates to the final ones is slow, while for highly stochastic components the convergence is more rapid but with larger revision errors (trade-off between stability and convergence).

2.3 STS model based decomposition

Time series components in a STS-model

The biggest difference between a Structural Time Series model and an ARIMA-based model, such as TRAMO-SEATS or X-12-ARIMA is in the formulation of the unobserved components. While the components, such as trends and cycles, do not have a direct interpretation in an ARIMA-based model, in a STS model this interpretaton is straightforward and direct.

An ARIMA-based decomposition requires a preparatory step including reg-ARIMA- or TRAMO procedures to clean the data from irregularities. In this step differencing of time series to achieve stationarity is almost always imposed resulting in the loss of degrees of freedom. However, for some very noisy series the stationarity can not be achieved in this way, not even if differencing is performed several times. Hence, applying this approach would result in a relatively bad estimates of the so called de-noised series (the error from reg-ARIMA procedure). As this de-noised series is the one to be decomposed into the seasonal effect, the trend-cycle and the irregular component, such an approach which would in turn lead to a large uncertainty in the estimated components.

The STS models on the other hand do not suffer from the stationarity issues since a time series Y_t to be decomposed is directly formulated as the sum of the above mentioned components. Hence, differencing to achieve stationarity is not necessary. Furthermore, the STS models do not require forecasting to obtain the end-point estimates which is an important advantage over the ARIMA-model based methodology. In principle, a univariate STS model may be viewed as a regression model where the explanatory variables are components from the classical decomposition model for a time series Y_t, as formulated here

$$Y_t = T_t + S_t + I_t\,, \qquad t = 1,\ldots,T, \tag{29}$$

where T_t is trend, S_t is seasonal component and I_t is irregular component. The explanatory variables are thus functions of time and the parameters are time-varying. As an STS-model may be expressed in many different and complex ways, the first step in the analysis is to find out which modeling alternative is most suitable for a particular time series or a set of time series. This issue is crucial and may appear similar to ARIMA-model based methodology. However, the difference between ARIMA-models and STS-models in this context is big: the STS-modeling framework does not need de-noising of original series in order to obtain a de-linearised series of errors to be decomposed into the basic time series components, which is needed for the reg-ARIMA (TRAMO) part of the ARIMA-model based procedures. Instead, the decomposition is applied directly to the original series, which is treated as a dependent variable. Hence, the pre-treatment step in the STS-models is reduced to find out a plausible modeling alternative within this framework. Once this choice is made the estimation of both the unobserved time series components and the possible other explanatory variables (e.g., calendar factors) is done in one single step. The decomposition is made as an integrated process through a *state space* form where the state of the system is represented through the unobserved components, such as trend-cycle and seasonal components (for details about the state space models see, e.g., Durbin and Koopman, 2001).

Basic Structural Model

Here is given a brief description of the basic STS-model and its components. See, e.g., Harvey (1990) for a more sophisticated description.

In its most basic form a STS-model may be formulated as follows

$$y_t = \mu_t + \gamma_t + \varepsilon_t\,, \qquad t = 1,\ldots,T, \tag{30}$$
$$\varepsilon_t \sim i.i.d.\ N\left(0,\sigma^2\right),$$

where μ_t, γ_t and ε_t are the trend, seasonal factor and irregular component, respectively. The expression (30) is called the basic structural model (BSM). All components are stochastic and each one is modelled separately.

The random error ε_t is usually called the *irregular component* in the seasonal adjustment literature. In the model's basic form this component is assumed to be a purely Gaussian white noise process, as indicated in (30). This implies that this component is modelled as a sequence of independent, identically distributed zero-mean random variables. Anyhow, the normality property is not exclusive since the irregular component might be modelled in different ways through a more complex modelling alternative. For simplicity, we focus on the basic form of a structural model where the irregular component is modelled either as Gaussian white noise or an ARIMA process (for details see, e.g., Harvey and Shephard (1993).

Extensions of BSM

Usually a BSM is extended to include the cyclical component and the predictor effects

$$y_t = \mu_t + \gamma_t + \psi_t + \sum_{j=1}^{m} \beta_j \mathrm{x}_{\mathrm{jt}} + \varepsilon_{\mathrm{t}}\,, \qquad t = 1,\ldots,T, \tag{31}$$

where ψ_t is cycle while the regression term

$$\sum_{j=1}^{m} \beta_j \mathrm{x}_{\mathrm{jt}}$$

incorporates effects of fixed regression coefficients that are likely to have influence on the response variable y_t.

Interventions may be included as regression effects as dummy or pulse variables, as explained in Harvey (1990, pp. 397-399). This approach is similar to RegARIMA approach but also allows for some extensions, for example, treating of changes in seasonal pattern.

Modelling the trend component

As mentioned earlier, each unobserved component may be modelled in a different way. As trend-component is defined as the natural tendency of a series in the absence of any noise (seasonality, effects of exogenous variables and unexplained variation expressed in the irregular component) it is natural to start with determining a good general model by modelling the trend in an optimal way. Two most common models for the trend component are the random walk (RW) model and the locally linear time trend (LLT) model. The RW model may be described as a model where the trend movement depends on the variance of the error term:

$$\mu_t = \mu_{t-1} + \eta_{\mathrm{t}}\,, \qquad \eta_{\mathrm{t}} \sim i.i.d.\, N\left(0, \sigma_\eta^2\right). \tag{32}$$

If this variance (σ_η^2) is zero then the trend is simply a constant.

The LLT model involves both the level and the slope in the trend representation:

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_{\mathrm{t}}\,, \qquad & \eta_{\mathrm{t}} &\sim i.i.d.\, N\left(0, \sigma_\eta^2\right) \\ \xi_t &= \beta_{t-1} + \xi_t\,, & \xi_t &\sim i.i.d.\, N\left(0, \xi_t^2\right) \end{aligned} \tag{33}$$

In (33) the disturbances η_{t} and ξ_t are assumed to be independent of each other and also independent of the main error ε_{t} in (31). The stochastic slope β_t follows a random walk model.

Expansions of these two basic models for trend are possible but this is usually not needed.

Model for cyclical component

The cyclical component (cycle) is treated as either deterministic or stochastic, depending on how the model is specified. A cycle is usually represented by period, amplitude and phase. A deterministic cycle assumes time-invariant amplitude and phase during the consecutive fixed periods meaning that the cyclical variations are repetitive and predictable. A model with stochastic cycle, on the other hand, is motivated by the fact that the cyclical variations usually vary over time influenced by random disturbances.

A deterministic cycle as a function of frequency λ, which is measured in radians, is expressed as a mixture of sine and cosine waves. This cycle depends on two parameters α and β , as shown here

$$\psi_t = \alpha \cos(\lambda t) + \beta \sin(\lambda t). \tag{34}$$

If t is measured on a continuous scale the amplitude will be $\omega = (\alpha^2 + \beta^2)^{1/2}$ and the phase is $\phi = \tan^{-1}(\beta / \alpha)$. This will lead to an equivalent formulation of the cycle in terms of the amplitude and phase as

$$\psi_t = \gamma \cos(\lambda t - \phi). \tag{35}$$

In most applications this pure deterministic form is not used. Instead, the cycle is usually built up recursively as a sum of cycles of different frequencies and amplitudes. This formulation leads to a stochastic cycle model of the following form

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos \lambda_c & \sin \lambda_c \\ -\sin \lambda_c & \cos \lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} v_t \\ v_t^* \end{bmatrix} \tag{36}$$

For more information about the properties of the specification and parameters in (36) see, e.g., Harvey (1990, pp. 38-39).

Modelling seasonal component

Seasonality in the context of STS-models is modelled in a way that allows for correction to the general trend of the series due to the periodic variations within a year. The simplest representation is a model of deterministic seasonality with the seasonal effect coefficients γ_t sum up to zero over a year. This model is described by the following expression

$$\gamma_t = \sum_{j=1}^{s} \delta_{jt} \gamma_t + \omega_t, \qquad \omega_t \sim i.i.d. N(0, \sigma_\omega^2), \tag{37}$$

where s is the number of seasons in the year, ω_t is a random disturbance term with zero mean and variance σ_ω^2 and the dummy variable δ_{jt} is equal to one in season j, zero otherwise. This model may be extended to have coefficients that may also change over time which would lead to a model for stochastic seasonality. One such model is the model where each seasonal effect is modelled as a random walk process, as follows

$$\gamma_t = \gamma_{j,t-1} + \omega_{jt}, \qquad \omega_t \sim i.i.d. N(0, \sigma_\omega^2), \quad j = 1, \ldots, s, \tag{38}$$

where the requirement that the seasonal components always sum to zero is accomplished by the restriction that the disturbance term sum to zero at each point in time.

Instead of dummy variables a model for seasonality may involve a set of trigonometric terms in a way similar to cycle representation in (36). A fixed seasonal pattern can also be modelled by a set of trigonometric terms at the seasonal frequencies $\lambda_j = 2\pi j / s, \quad j = 1, \ldots, [s/2]$, where $[s/2]$ implies rounding down to the nearest integer, leading to the following expression

$$\gamma_t = \sum_{j=1}^{[s/2]} \left(\alpha_j \cos \lambda_j t + \beta_j \sin \lambda_j t \right) . \tag{39}$$

A seasonal pattern in (39) may be allowed to evolve over time in a similar manner as the stochastic cycle in (36) which would lead to different extensions of the basic models for seasonality. See, e.g., Harvey (1990, pp. 40-42) or Harvey and Shephard (1993) for more details.

Modelling the Irregular Component

The structural dynamics of a response series y_t is captured by the previosuly explained components, such as trend, cycle, seasonal and regression effects. Hence the irregular component represents the unexplained remaining part in the series which corresponds to residual variation in an ordinary regression model. This residual variation might be treated in different ways from a very restrictive representation such as Gaussian white noise to far more complicated structures.

Statistical treatment: estimation, decomposition and diagnostic checking

In statistical sense the STS-models are usually treated through a general state space representation by using the Kalman filter algorithm. This generalisation allows treatment of both linear and non-linear form of a STS-model. Usually, a linear representation will be typical in the practical work since non-linear extensions are generally difficult to handle because of a huge variety of possible model specifications. An introduction to the linear and non-linear state space models is given in Durbin and Koopman (2001, Ch. 3 and Ch. 10, respectively).

The general linear Gaussian state space model for a time series y (or a set of time series $\mathbf{y}$ with N elements) consists of a measurement equation and a transition equation, respectively:

$$y_t = Z_t \alpha_t + X_t \beta + \varepsilon_t , \tag{40.a}$$

$$\alpha_t = T_t \alpha_{t-1} + W_t \beta + R_t \eta_t . \tag{40.b}$$

See, e.g., Harvey and Shephard (1993, pp. 267-268) for details about (40). The observable variable y_t is related to a state vector α_t whose elements are not observable. However, the observations carry some information which can be estimated. This estimation is typically done by the Kalman filter, which is a recursive procedure for computing the optimal (in terms of the minimum mean square error) estimator of the state vector at time t. Hence, the state vector contains information about the unobserved components of time series y_t, such as seasonals, trend and irregulars. The estimation of all parameters is performed by the maximum likelihood method via the prediction error decomposition. Hence, the likelihood is evaluated by the Kalman filter using a numerical optimisation method for maximisation of likelihood.

See, e.g., Durbin and Koopman (2001, Ch. 2, 4 and 5) for details about the Kalman filter.

Since the state space form of structural models for time series is a model-based maximum likelihood approach it has many desirable statistical properties. As noted earlier, model selection does not rely on correlograms and related statistical devices in the way that the ARIMA model-based procedures do, which would imply differencing to obtain stationarity. This basically means that the variables and components are estimated in levels, which is an advantage in terms of interpretation of the estimated components.

The maximum likelihood approach within a state space framework provides a vehicle to make inference about the estimated components. It is relatively easy to make forecasts for each component with associated forecast errors since the mean square errors may be computed.

Hence, the most important step is actually the model selection with respect to modelling of each component in the general structural model. Harvey (1990, p. 13) discusses the most important criteria for a good modelling approach:

a) Parsimony – a simpler model should be preferred to a more complicated one meaning that a model with a relatively small number of parameters should be preferred to a model with large number of parameters.

b) Data coherence – the chosen model should provide a good fit to the data and the residuals should be approximately random.

c) Consistency with prior knowledge – if there is any relevant information in economic theory or from any other relevant sources the model should be consistent with this information.

d) Data admissibility – natural restrictions should be reflected in the model's ability to estimate and predict, e.g., model estimation for the variables that cannot be negative should not produce any negative value.

e) Structural stability – good fit outside the sample is required.

f) Encompassing – if a model is able to explain the results given by the rival formulation then it is said to encompass a rival formulation. This means that a rival model does not contain any information which could be used to improve the chosen model.

Once a plausible model is chosen the application of maximum likelihood and Kalman filter is straightforward but the technical details are less important in this context.

After estimation the diagnostic checking may be performed by using significant tests, usually based on three main assumptions concerning the residuals in the linear Gaussian state space models. These assumptions are independence, homoscedasticity and normality, which correspond to the general assumptions for a linear regression model. This is diagnosed by utilising the standardised prediction errors, as explained in, e.g., Commandeur et al. (2011, p. 9) or in Harvey and Koopman (1992).

Motivation for the use of STS- models

The two standard methods, TRAMO-SEATS and X-12-ARIMA, are widely used in official statistics and generally recommended by the European authorities. The reasons behind their popularity are natural. First of all these methods are relatively easy to interpret and implement in the statistical production since they are widespread across many well-supported IT-platforms. Furthermore, these methods have all necessary facilities that a modern seasonal adjustment procedure requires. Usually, these two methods perform well in terms of short-run forecasting which makes them attractive to the policy makers.

However, when certain relatively strong assumptions for the underlying time series are not satisfied these modelling alternatives are likely to produce poor estimates of the related components. This is particularly true for the time series contaminated by many aberrant observations, the time series with strong moving seasonality or the data with other evident non-linearities. In some cases, the STS-models might be helpful since this framework may utilise varying coefficients for the moving seasonality problems or non-parametric methods (splines) to deal with non-linearities.

The usual assumption for a basic structural model is that the variance of each component is kept constant but it is possible to create a more complex extension which allows the trend component to be dependent on the business-cycle. The time-varying confidence intervals for seasonally adjusted estimates may be created in this way, as proposed in Koopman and Franses (2001).

One important issue regarding a standard seasonal adjustment from an ARIMA model-based procedure is how to treat calendar correction, especially with respect to estimation of moving holidays (such as Easter). Generally, the estimated parameters are held fixed as a result of ordinary least squares- or related estimation procedure. The state space framework permits these effects to vary over time which is practically impossible in the case with the competing methods. An example of a structural model involving the stochastic trading-day variations within a month is given in Dagum and Quenneville (1992).

Concerning the non-linear and non-Gaussian state space models, a detailed overview may be found in Durbin and Koopman (2001, Ch. 10). The STS-models within a state space framework can also tackle problems with temporal aggregation which is usually treated by a benchmarking procedure, as proposed in, e.g., Durbin and Quenneville (1997).

Furthermore, the structural state space models allows for a treatment of observations sampled at a higher frequency than monthly, meaning that the weekly, daily or even hourly observations can be treated within this framework. This is practically impossible with the two main competitors, TRAMO-SEATS or X-12-ARIMA. See, e.g., the study about the estimation of weekly seasonal pattern for the UK money supply in Harvey, Koopman and Riani (1997) or the estimation of hourly electricity data in Harvey and Koopman (1993). A treatment of different data irregularities, such as missing observations and observations at mixed frequencies, is illustrated in the study by Harvey and Cheung (2000) on the measurement of British unemployment.

Furthermore, if there is existence of complex relationships among different variables in a system of time series, a multivariate framework might be an alternative to a traditional univariate seasonal adjustment. Neither TRAMO-SEATS nor X-12-ARIMA have possibilities to treat multivariate time series. On the other hand, the univariate STS-models may relatively easily be extended to a multivariate framework. An overview of available software for state space models including an introduction into multivariate structural framework is given in Commandeur, Koopman and Ooms (2011). Different extensions to cope with more specific problems in a multivariate framework are described in, e.g., Koopman and Durbin (2000), Casals, Jerez and Sotoca (2002) and Birrell, Steel and Lin (2008).

The STS-models may also be extended to tackle the estimation problems with repeated overlapping sample survey. Pfeffermann (1991) proposes statistical treatment within this framework for estimation of population means based on rotating panel surveys when these surveys are overlapping. The proposed model allows for changes over time that might arise from an increase in sample size or a change in survey design. This framework permits a natural extension from a univariate STS-model to a multivariate STS-model, as described in Harvey and Shephard (1993). A monograph by Birrell (2008) gives a detailed description of a multivariate state space model tailored to the situation where a seasonally adjusted aggregate series is constructed by jointly modelling a set of sub-series.

Some conclusions

Obviously, the STS-models belong to a comprehensive framework suitable to almost any kind of time series analysis. Any ARIMA-model may be expressed in terms of a STS-model but the STS-models are much more than extensions of an ARIMA-modeling framework. They may involve State-Space approach, Bayesian approach, multivariate seasonal adjustment, non-linear models etc. The STS-models can handle data with irregular structure, the data with missing values and they are likely to be robust to different misspecifications.

Such flexibility may look attracting but these models have not been extensively used in official statistics. One reason for this is complexity of different modelling alternatives within this framework. Commonly, the methodological competence of an ordinary user at a national statistical office is rarely on a level required to understand the theoretical issues behind the procedures. Furthermore, the main-stream methods are likely to perform well for a large number of time series which quite naturally motivate for their use. And finally, complexity is not always easy to handle – not even for a specialist.

Anyhow, in some cases when the recommended procedures are not flexible enough to handle some deviations from the major assumptions they rely on, the STS-models might be helpful.

2.4 Step by step seasonal adjustment

The method of seasonal adjustment consists of several theoretical and practical issues which should be considered during the procedure in order to meet the expectations of experts and users. Although the modules “Seasonal Adjustment – Introduction and General Description” and “Seasonal Adjustment – Issues on Seasonal Adjustment” provide a comprehensive summary concerning the details, it is also necessary to describe the exact steps of the adjustment.

The following figure summarises the steps which are detailed in this subsection:

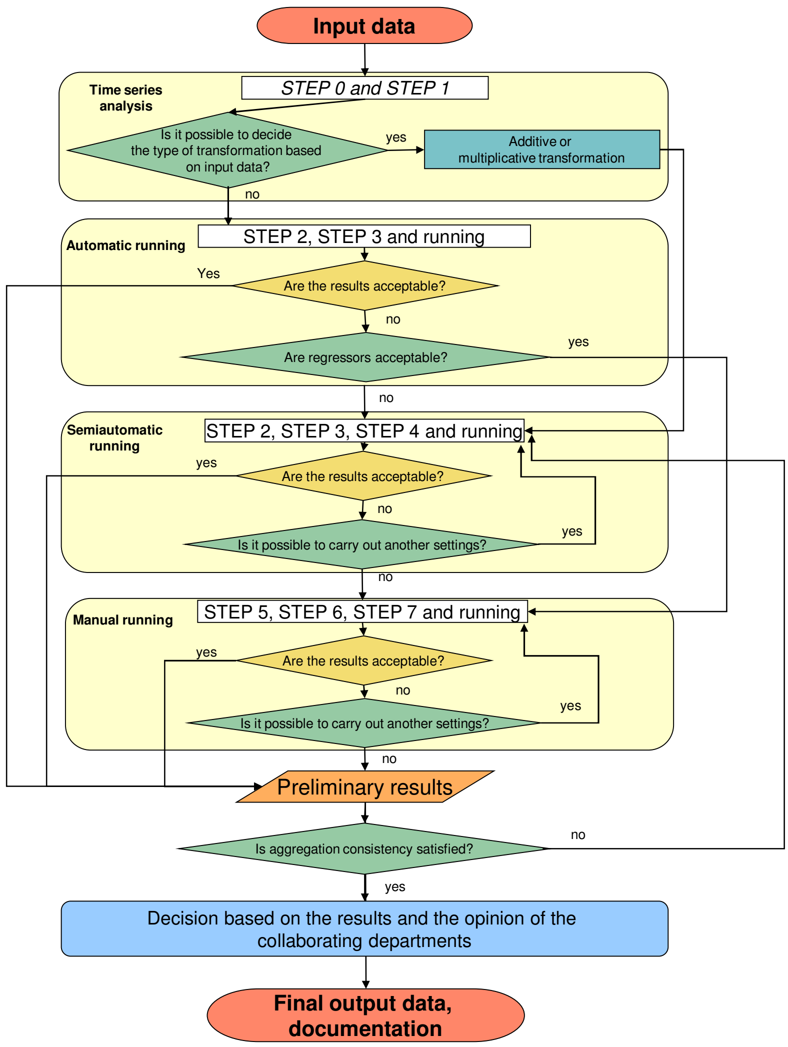

Figure 7: Steps of seasonal adjustment (source: HCSO)

STEP 0 – Examination of basic conditions and collection of expert information

Before the seasonal adjustment of a time series for the first time or the revision of the model and parameters, some basic properties of the given time series should be examined in order to achieve an adequate result:

- It is a software requirement (for both TRAMO-SEATS and X-12-ARIMA) for seasonal adjustment that the time series have to be at least 3 year-long (36 observations) for monthly series and 4 year-long (16 observations) for quarterly series. Naturally, these are minimum values; series can be longer for an appropriate adjustment. Series shorter than 3 years should not be seasonally adjusted by standard procedures, but in case of alternative, less standard procedures, it is possible (Hood, ECB (2003), EC (2005)). Special attention is necessary if series are 3-7 year-long as a result of instability problems. In this case, a general rule is to check the specification of the parameters several times per year (ESS guidelines). It is important to inform users about instability problems for short time series. However, if the time series is very long, the seasonal adjustment does not necessarily lead to higher quality because seasonality can change as time goes on. The sources of changes are the change in concepts, definitions, methodology, legislative events, change of the weather, etc. If the series are not consistent for some reason, it might be better to shorten them for the purpose of identifying a more consistent seasonal pattern and to improve the decomposition. Another option in treating inconsistencies is to provide two separate time series, one for the latest period and one for an earlier period.

- Missing observation(s) in the time series should be identified. The identification is carried out, for example, via graphical analysis. Too many missing values in the given series lead to estimation problems in the adjustment. Thus, statisticians should substitute the missing observations with alternative data or statistical methods in the lack of original data.
- If series are part of an aggregate series, it should be verified that the starting and ending dates for all component series are the same.

If the aforesaid conditions hold, then preliminary expert information has to be collected about the

- calendar effects (trading/working day, leap year, moving holidays (e.g., Easter), national holidays)
- outliers
- seasonality
- methodological change of specialisation statistics
- methodological change of exterior factor (e.g., law, order)

Expert information is important, especially if the diagnostics of the adjustment are inconclusive (for example, outlier detection at the end of a time series) or in case of manual decomposition.

STEP 1 – Time series graphical analysis

Graphical analysis of the original time series provides useful information to the analyst because visual graphs help in identifying possible problems, quality issues and give relevant information to the process of seasonal adjustment.

Basic graphics

There are **basic graphs** by which possible problems in the data (such as outliers, zeros, negative values, missing observation(s) etc.), the structure of the trend-cycle or of the seasonal component are revealed or the presence of seasonality is examined.

Seasonality in a time series can be identified by regularly spaced peaks and troughs which have a consistent direction and approximately the same magnitude every year, relative to the trend. The presence of seasonality is pre-condition of seasonal adjustment. Figure 8 illustrates a clear seasonal pattern.

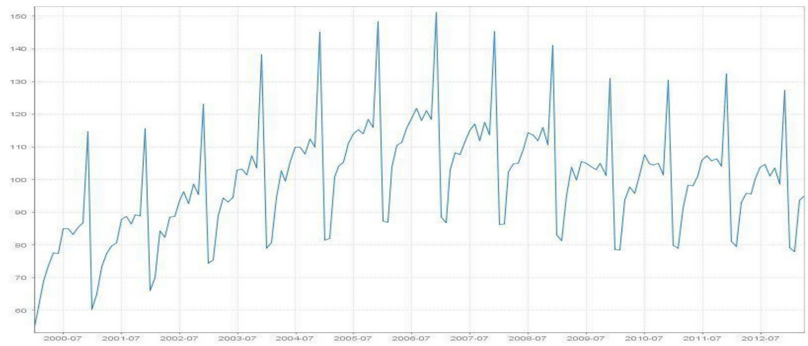

Figure 8: Hungarian monthly retail volume index, original series (source: HCSO)

Outliers could strongly affect the quality of the seasonal adjustment. The impact of these abnormal values could distort the estimation of components, therefore the seasonally adjusted series and the trend (the two most published and important data about seasonal adjustment) as well. In this part of the analysis, outlier identification and verification are carried out by addition of basic graphs and expert information. For example, if the graph of the original time series shows abrupt changes or there are data which do not fit in the past behaviour of the series, statistician should examine if these phenomena are valid (so they refer to the presence of outliers), or there are sign problems in the data, for example, captured erroneously. The two circled data do not fit in the past behaviour of the series. In such case, if there is an economic explanation behind the changes, data can be outlier.

The type of decomposition should be used automatically. Besides, there are situations when the diagnostics for choosing between decomposition schemes (models) are inconclusive. In this case one can choose to continue with the type of decomposition used in the past to allow for consistency between years, or if there is no experience about the past it is recommended to visually inspect the graph of the series.

- If the series has zero and negative value(s), or if the difference of the trend and the observed data is nearly constant in similar periods of time (months, quarters) irrespectively of the tendency of the trend, additive model is needed.
- If the series has a decreasing level with positive values close to 0, multiplicative model is considered.

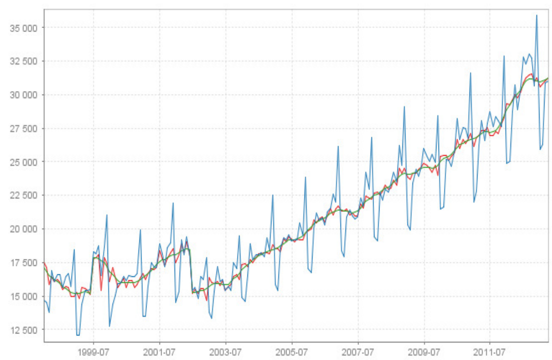

Figure 9: Additive decomposition

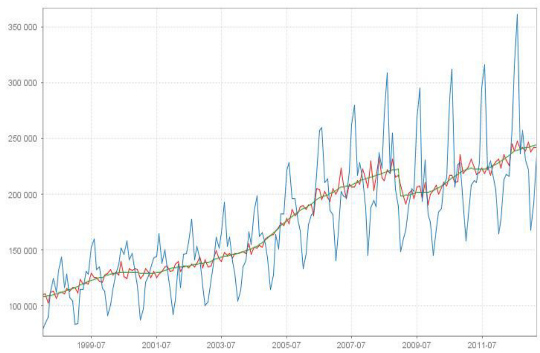

Figure 10: Multiplicative decomposition

Alternative approaches

Besides, there are more **sophisticated graphs**, such as spectrum or autocorrelograms which are two important tools of detecting seasonality and trading day effects in a time series. The peaks appearing in the spectrum indicate periodicity in the time series corresponding to the given frequency. Some frequencies are more important than others:

- *seasonal frequencies* show how many cycles of phenomenon are per year. For example, for monthly series the seasonal frequencies are (a whole period is represented by π): $\pi/6$, $\pi/3$, $\pi/2$, $2\pi/3$, $5\pi/6$, which are equivalent to 1, 2, … cycle per year. Peaks at the seasonal frequencies indicate the presence of seasonality. Seasonality is a precondition for seasonal adjustment.

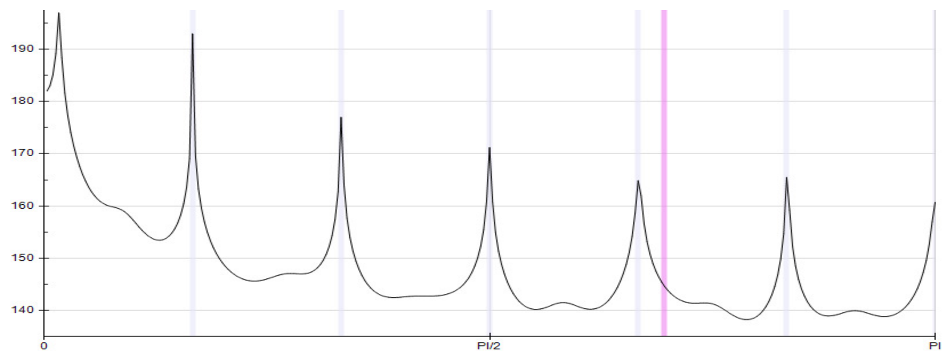

Figure 10: Auto-regressive spectrum of time series. Clear peaks at frequency $\pi/6$ and its multiples.

- peaks at trading days frequencies could occur due to inappropriate regression variables used in the model or the significant change of the calendar effect because the calendar effect cannot be modelled by fixed regression effect on the whole time series span.

Autocorrelation is the cross-correlation of a time series with itself. It is a mathematical tool for finding repeating pattern, to detect non-randomness in data, such as the presence of seasonality. In an autocorrelogram only positive and statistically significant autocorrelation at seasonal lags is important because of the concept of seasonal fluctuation. Figure 11 shows autocorrelogram of monthly time series. It is clear to see the significant autocorrelation at seasonal lags (12 and its multiples). In contrast with autocorrelogram, the partial autocorrelogram does not give reliable information about the presence of seasonality; its usefulness is to identify the ARIMA model.

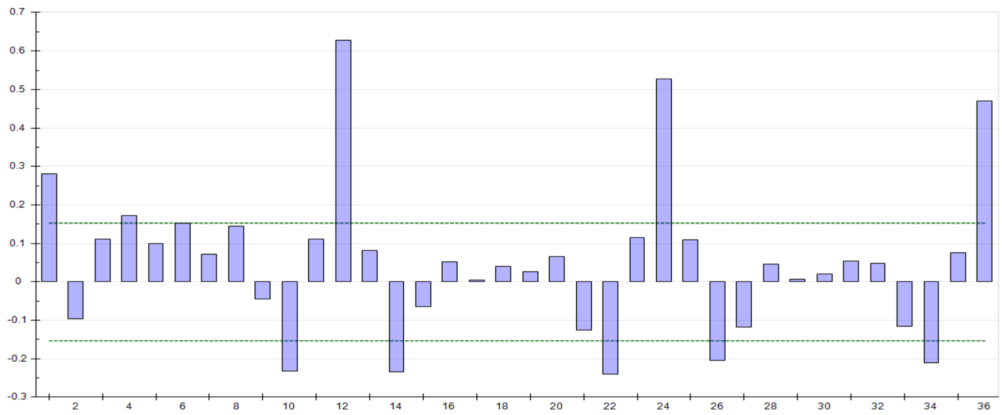

Figure 11: Autocorrelogram

The time series graphical analysis can be carried out by Eviews, R, SAS, Demetra+, JDemetra+, etc.

STEP 2 – Transformation

When the variance of the given time series is not constant, the series should be transformed in order to achieve stationary autocovariance function; hence, it stabilises the variance of the original time series. There are several ways of the transformation such as taking the logarithm, square root or differencing. The most commonly used is taking the logarithm. Log-transformation is offered by both TRAMO-SEATS and X-12-ARIMA. These software operate with automatic test which helps the user to choose between transformation types:

- no transformation → additive model is considered;
- log-transformation → log-additive model is used.

Confirm the results of the automatic choice by looking at the graphs of the series as it is described in STEP 1.

STEP 3 – Calendar adjustment

Calendar adjustment can be executed in a number of ways. One can distinguish between proportional and regression methods for adjustment. Under proportional approach the effects of trading days are estimated by counting the proportion of them on the month/quarter. Under the regression approach the effects of trading days are estimated in a regression framework. If possible, the proportional approach should be avoided – especially in case of model-based methods. The most recent and widely used seasonal adjustment tools (TRAMO-SEATS, X-12-ARIMA, X-13-ARIMA-SEATS) perform calendar adjustment by regression method, called reg-ARIMA. Under the reg-ARIMA approach it should be determined which regression effects (trading/working day, leap year, moving holidays) and national holidays are plausible for the series.

If an effect is not plausible for the series or the coefficients for the effect are not significant, then regressor should not be fit for the effect, it should be eliminated. Exception can be made in case of trading day regressors (see 2.1.1.).

If the coefficients for the effects are marginally significant then it should be determined if there is a reason to keep the effects in the model. For example, if there are some kinds of economic explanations behind the effects, they should be retained.

It is important to distinguish between seasonal and non-seasonal component of calendar effects, since the seasonal part of calendar effects is eliminated by the seasonal adjustment filters under the decomposition procedure of time series (see STEP 6). Therefore, under calendar adjustment within the pre-treatment of seasonal adjustment only the non-seasonal part of the effects has to be dealt with.

Seasonal adjustment approaches of the Demetra software family (TRAMO-SEATS, X-12-ARIMA, X-13-ARIMA-SEATS) automatically create appropriate calendar regression variables depending on the chosen specification. However, the user may need to change the automatic options, for example, for chaining two calendars for two different time periods or modifying the calendar regression variables to match the national holidays which differ from the previous options of the used software. Sometimes the automatic test does not indicate the need for trading day regressor, but if there is a peak at the first trading day frequency of the spectrum of the residuals, then one may fit a trading day regressor manually.

STEP 4 – Outliers

The presence of these abnormal values distort of the seasonal and calendar components because seasonal adjustment methods are usually based on linear models (e.g., reg-ARIMA). Therefore, outliers should be identified and removed before seasonal adjustment is carried out. Besides, they give information about some specific events (like strikes, etc.), so valid outliers should be reintroduced after the adjustment.

There are two possibilities to identify outliers. The first is when we identify series with possible outlier values by looking at graphs of the original series and any available information (economic, social, etc.) about the possible cause of the detected outlier, as in STEP 1. Since seasonal adjustment is carried out by software in practice, this direction is in service as an additional opportunity generally to the cheque of the automatic outlier detection. Therefore, the second possibility what we use is automatic outlier detection and correction. Outlier detection is always carried out automatically when time series are seasonally adjusted for the first time.

Outlier coefficients may be statistically non-significant when time series are already seasonally adjusted and reg-ARIMA models are revised (generally once in a year). In this case, the user has to decide whether to keep them in the model. There are criteria, for example, coherence with past decisions, based on which we may come to our decision..

The reliability of the seasonal adjustment depends on the number of outliers. A large number of outliers relative to the length of the series could result in over-specification of the regression model. Furthermore, it signifies if there is a problem related to weak stability of the process, or if there is a problem with the reliability of the data (for example, data captured erroneously). Shortening the time span or changing the critical value of the statistical tests may help in better modelling of outliers.

It is important to stress the treatment of outliers at the end of the series. For example, the change of the type of outlier later may lead to large revisions. In this case expert information is especially important because the type of outliers at the end of the series are uncertain, as real extraordinary economic effects are often unknown, and there is no information on what happens after the latest outlier appears. For instance, the level shift is indistinguishable from an additive outlier in this case, since we do not know how the level of the series will behave. Therefore, to collect external information on the event in question is very useful. It would help to define the type of outliers at the end of the series.

STEP 5 – ARIMA model

In the most widely used software – TRAMO-SEATS and X-12-ARIMA – seasonal adjustment are based on ARIMA-model methodologies. Automatic model identification usually produces satisfactory models. But there are cases when results are not plausible. Therefore, manual identification may be justified. Another situation when different ARIMA models could fit in the same series. In this case it is recommended by most of the statisticians to choose the simplest model with the smallest number of parameters with a satisfactory fit. This is better than a high-order model. During manual procedure, it is advisable to identify the not significant high-order ARIMA model coefficients and reduce the order of the model, taking care not to skip lags of autoregressive models. For moving average models, it is not necessary to skip model lags whose coefficients are not significant. Before choosing an MA model with skipped lag, the full-order MA model should be fitted and skip a lag only if that lag's model coefficient is not significantly different from zero.

Another situation in which manual identification may be justified is when automatic identification produces a model which, while satisfying the tests, still has some unsatisfactory features. For example, some individual significant correlation at fairly low lags, although the combined test on the serial correlations of the residuals passed. In this case it could be worth adding an extra coefficient at the appropriate lag to the AR or MA component. If the extra coefficient is significant and the significant serial correlation has been removed, the extra term may be justified.

The model identification statistics, particularly the BIC and the AIC, are useful tools in confirming the global quality of fitting statistics. The application of information criteria may help in choosing among different models.

STEP 6 – Decomposition

The last step of the pre-treatment procedure for seasonal adjustment is to decompose the original time series into different components: trend-cycle, seasonal and irregular component. Depending on the nature of seasonality components, several different schemes can be connected. The most frequently used schemes (models) are the following:

- *additive decomposition* (Figure 9), when the magnitude of seasonal effects does not change as the level of the trend-cycle changes. Also, any series with zero or negative values are additive. In this case, components are linked additively.
- the *multiplicative decomposition* implies that as the trend of the series increases, the magnitude of the seasonal spikes also increases (Figure 10). For multiplicative decomposition, components are linked through multiplication. The decomposition scheme of the most economic time series is multiplicative.
- *log-additive* scheme is to specify an additive model on the logarithm of the time series. Based on this fact, one of its main advantages is that the multiplicative model can be transformed to additive model, which is more manageable. Therefore, multiplicative and log-additive model are frequently considered identical.

Before the decomposition of a time series, some modifications should be carried out on it. It is required to determine and remove deterministic effects such as outliers or the non-seasonal part of calendar effects, because the adjustment is distorted in case of their presence. After removing the deterministic part we get the purely stochastic part of the series. This is the autocorrelated disturbance of the deterministic part. It is decomposed by filters based on linear stochastic models. We get the final component of the series, if regression effects (deterministic part of the series) are reintroduced in the components according to their nature.

STEP 7 – Quality diagnostics

The procedure of seasonal adjustment is very complex, so the accurate monitoring of the results before disclosure and disseminated is very important. A wide range of quality measures are available to ensure the best quality.

Quality diagnostics of seasonal adjustment can divide into three main parts. The tree issues are the following:

- model adequacy and diagnostics on the model residuals
- residual trading day effects and seasonality in both the seasonally adjusted series and the irregular component
- stability analysis

In the first part of monitoring the results it can be examine if the model used for the adjustment is adequate. At ARIMA modelling, the principal tool for assessing model adequacy is the widely-used Ljung-Box statistic, built from autocorrelations of residuals. Of particular interest are autocorrelations at low lags, say 1 to 4, and at seasonal lags 12 and 24. Low Ljung-Box p-values (below .05) at lags 12 and 24 result from one or more high residual autocorrelations and indicate model inadequacy. Monitoring of the seasonal moving average parameter is also important. When it is close to -1, the seasonal factors are highly stable; when it is close to 0, the factors tend to change rapidly. Series graphs and knowledge of the series can help assess how much movement in the seasonal is desirable.

After seasonal adjustment, we can check for residual seasonality and residual calendar effects using spectral graphics of the decomposed seasonally adjusted series and the irregular component. Peaks at seasonal frequencies of the adjusted series mean that the filters used in the decomposition are not well adapted to the series or to a large part of it. Peaks at the trading day frequencies could indicate that the regression variables of the model do not suite well the series or that the calendar effects change too much to be captured by the fixed regression effects applied for the whole duration of the series. If remaining seasonality is present one has to reconsider the model specification, the regression variables or the time span used for modelling.

Careful assessment of the seasonally adjusted data includes analysis about the stability of the seasonal component. The software report several stability diagnostics such as statistical tests or graphical diagnostics. Revision history and sliding spans are the most commonly used stability diagnostics.

Revision history analyses what kinds of revisions are caused by adding new observations at the end of the series. It presents charts both for the seasonally adjusted and trend-cycle series. On Figure 12 each circle depicts the initial adjustment when this point is the last observation. The curve presents the final results. The closer the initial observation dots to the curve based on all available observations, the better the quality. Revision history table is also available in this part. This table presents the differences between the first estimates and the last estimates for the last four years. If some observations are exceeded the given critical limit, it should be examined whether the adjustment is unsatisfactory or these abnormal values are, in fact, outliers.

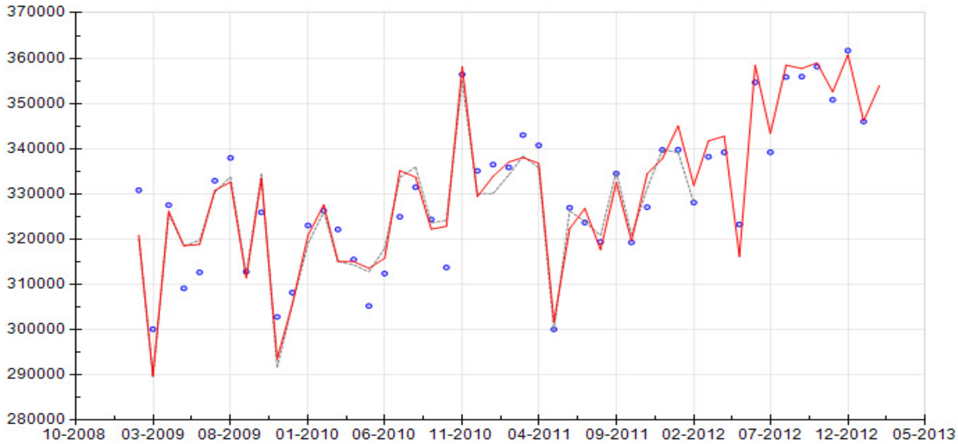

Figure 12: Revision history

Another very important tool for stability analysis is the sliding spans. It is particularly useful for a series with a large number of outliers or changes in seasonality. It depicts period-to-period changes. The results are stable if one cannot consider values exceeding a three per cent threshold. Any larger value is unstable. Figure 13 shows stable seasonal factors since none of the values exceeds three per cent.

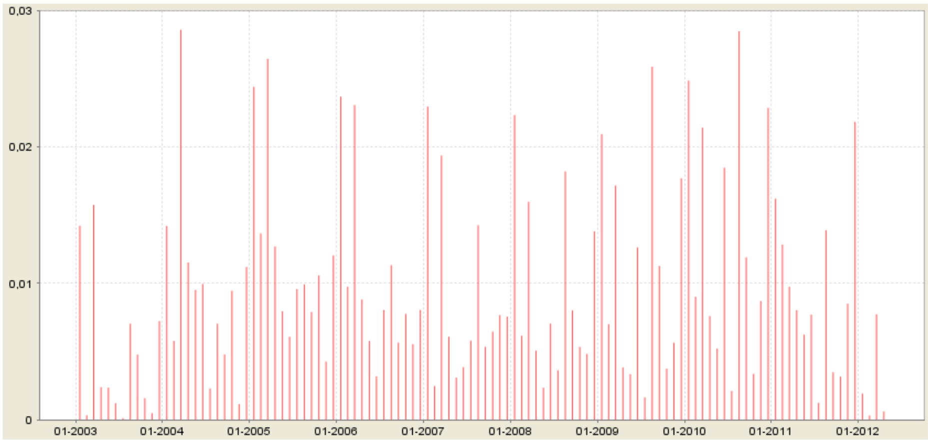

Figure 13: Sliding spans analysis

3. Preparatory phase

4. Examples – not tool specific

5. Examples – tool specific

6. Glossary

For definitions of terms used in this module, please refer to the separate "Glossary" provided as part of the handbook.

7. References

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Specific section

8. Purpose of the method

The method discusses the theoretical background of seasonal adjustment with a comprehensive summary of methodological principles and the steps of the adjustment process: the main focus of this module is put on description of the decomposition based on ARIMA models, on moving averages and on STS-models, while the other classes of models are not treated.

9. Recommended use of the method

1. ARIMA: A particularly important part of seasonal adjustment is the identification of ARIMA models. This tool, as discussed by Box and Jenkins (1976), represents a practical way of dealing with moving features of seasonal time series.
2. STS: Any ARIMA-model may be expressed in terms of a STS-model but the STS-models are much more than extensions of an ARIMA-modelling framework. They may involve State-Space approach, Bayesian approach, multivariate seasonal adjustment, non-linear models, etc. The STS-framework can handle data with irregular structure, the data with missing values and they are likely to be robust to different misspecifications.

10. Possible disadvantages of the method

1. ARIMA: An ARIMA-based decomposition requires a preparatory step including reg-ARIMA- or TRAMO procedures to clean the data from irregularities. In this step differencing of time series to achieve stationarity is almost always imposed resulting in the loss of degrees of freedom. However, for some very noisy series the stationarity can not be achieved in this way, not even if differencing is performed several times. Hence, applying this approach would result in a relatively bad estimates of the so called de-noised series (the error from reg-ARIMA procedure). As this de-noised series is the one to be decomposed into the seasonal effect, the trend-cycle and the irregular component, such an approach which would in turn lead to a large uncertainty in the estimated components.
2. STS: This method is really flexible and robust, but these models have not been extensively used in official statistics as a result of the complexity of different modelling alternatives within this framework. Furthermore, the main-stream methods are likely to perform well for a large number of time series which quite naturally motivate for their use. And finally, complexity is not always easy to handle – not even for a specialist.

11. Variants of the method

1. ARIMA
2. STS

12. Input data

The original time series before seasonal adjustment.

13. Logical preconditions

In this module, this point is not relevant.

14. Tuning parameters

Not relevant.

15. Recommended use of the individual variants of the method

It is discussed in point 9.

16. Output data

The output data contains the results of seasonal adjustment: the components of time series after decomposition and the elimination of irregularities, and the adjusted time series.

17. Properties of the output data

Not relevant

18. Unit of input data suitable for the method

Not relevant

19. User interaction - not tool specific

Not relevant

20. Logging indicators

Not relevant

21. Quality indicators of the output data

The quality indicators represent the adequacy of the seasonal adjustment process. A primary purpose is identifying the available best model. The statistical tests such as Ljung-Box and Box-Pierce tests offer the opportunity to examine the adequacy of the chosen model. The robustness is also essentially important, which may be studied via sliding spans.

22. Actual use of the method

Discussed in point 9 and 10

Interconnections with other modules

23. Themes that refer explicitly to this module

1. Seasonal Adjustment – Introduction and General Description
2. Seasonal Adjustment – Issues on Seasonal Adjustment

24. Related methods described in other modules

1.

25. Mathematical techniques used by the method described in this module

1.

26. GSBPM phases where the method described in this module is used

1. GSBPM Phase 6.1, 6.2, 6.3

27. Tools that implement the method described in this module

1.

28. Process step performed by the method

Administrative section

29. Module code

Seasonal Adjustment-M-SA of Economic Time Series

30. Version history

Version	Date	Description of changes	Author	Institute
0.0.1	01-02-2013	STS models	Suad Elezović	SCB Sweden
0.0.2	01-02-2013	step by step	Orsolya Kocsis	HCSO
0.0.3	02-04-2013	decomposition	Anna Ciammola	ISTAT
0.0.4	27-05-2013	ad hoc filters	Oyvind Langsrud	Statisics Norway
0.0.5	08-07-2013	reg-ARIMA	Anna Ciammola	ISTAT
0.0.6	02-09-2013	glossary	Anna Ciammola	ISTAT
0.0.7	18-09-2013	step by step	Orsolya Kocsis, László Sajtos	HCSO
0.0.8	20-09-2013	STS models	Suad Elezović	SCB Sweden
0.0.9	04-10-2013	new subsection 2.4 and other changes	Suad Elezović, Yingfu Xie	SCB Sweden
0.2	22-11-2013	specific section	László Sajtos	HCSO
0.2.1	11-12-2013	preliminary release		
0.3	20-12-2013	minor improvements based on EB-review	László Sajtos	HCSO
1.0	26-03-2014	final version within the Memobust project		

31. Template version and print date

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