This module is part of the

Memobust Handbook

on Methodology of Modern Business Statistics

26 March 2014

Method: Denton's Method

Contents

General section

1. Summary

The Denton method is a well-known method for benchmarking. Its aim is to achieve consistency between time series on the same target variables that are measured at different frequencies (for instance annual data with quarterly data) with a different reliability. Following the literature, these periods will be called annual and sub-annual periods, respectively. This terminology can be used without loss of generality, sub-annual and annual periods can be any combination of two different periods with unequal lengths, such that one annual period covers a whole number of sub-annual periods.

The method may be applied to time series, consisting of at least one annual period. In achieving consistency, the sub-annual data are adjusted, while the annual data are not changed (i.e., at least not in the method that is originally described by Denton in 1971). Furthermore, the Denton method attempts to preserve the sub-annual changes of the high-frequency data as much as possible.

Originally, the Denton method is defined for univariate data. However, in the literature a lot of extensions are described, for instance for the multivariate case.

Mathematically, the Denton method translates a data reconciliation problem into a weighted quadratic optimisation problem under linear conditions. As mentioned by Bloem et al. (2001) the Denton method is very well suited for large-scale applications.

2. General description of the method

Below we give a non-technical description of the Denton method. For a more comprehensive and a more technical description we refer to Denton (1971), Dagum and Cholette (2006), and Chen (2007). A reference for an extended version for multivariate data is Bikker et al. (2010).

The Denton method is a macro-integration method (cf. "Macro-Integration – Main Module") which is specially aimed at benchmarking. Therefore, the Denton method is more specific than other macro-integration methods, like RAS (see "Macro-Integration – RAS") and Stone's method (see "Macro-Integration – Stone's Method").

Benchmarking is the problem of achieving consistency between time series for the same target variables, that are measured at different frequencies (for instance: quarterly time series on the one hand and annual time series on the other hand). For flow variables consistency means that sub-annual values add up to annual values, for instance: the sum of four quarterly values being equal to one annual value (the so called *annual alignment*).

As mentioned by Bloem et al. (2001) it is imperative that sub-annual and annual time series are consistent. Inconsistent time series would confuse users and cause uncertainty about the actual situation. However, because data that are measured at different frequencies are collected by different methods, using different sample surveys and different data processing methods inconsistencies occur naturally, i.e., the annual figures are not the same as the annual sums of the underlying sub-annual data. Thus, benchmarking is an important process for statistical offices.

Typically, the annual data sources are of high precision and provide reliable information on the overall levels. Therefore the annual series are considered as benchmarks. On the other hand, the sub-annual data are less precise, but they provide the only information on the short-term movements. Benchmarking combines the (assumed) strengths of both types of data. The sub-annual time series are adjusted, to align with the annual benchmarks, while preserving as much as possible the initial changes between each two successive sub-annual periods.

The Denton method is a widely applied method for benchmarking. A distinguished feature of the Denton method is that it preserves as much as possible *all* initial changes of the sub-annual time series. This property is known as the *movement preservation principle* and is illustrated in Figure 1.

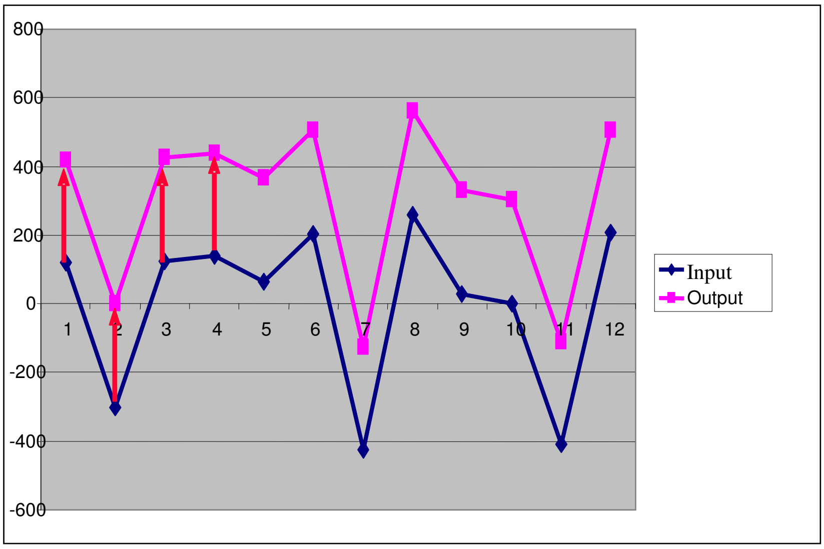

Figure 1. The movement preservation principle.

The movement preservation applies to whole time series, including the transitions from the last sub-annual period of one year to the first sub-annual period of the following year. Benchmarking methods that do not satisfy the movement preservation principle may suffer from the *step problem*, i.e., large gaps between two successive sub-annual periods in different years.

A straightforward, widely known benchmarking method that heavily suffers from the step problem is the *pro-rata method*. This method achieves consistency between annual and sub-annual time series by multiplying all sub-annual periods by correction factors. These correction factors are the so-called *proportional annual discrepancies* (Dagum and Cholette, 2006, page 53) and are defined by the ratio between an annual value and the sum of all underlying sub-annual values. Thus, the same factors are used for all sub-annual periods that belong to one year.

Figure 2 compares the results of a Denton method and the pro-rata method for a fictitious example. The example consists of 12 quarters. Initially, all quarterly values are 10. There are three annual benchmarks, which equal 100, 400 and 200 respectively. Then, the proportional annual discrepancy for the first year is: 100 / (10 + 10 + 10 + 10) = 2.5. Thus, the pro-rata method adjusts the first four quarterly values by a factor 2.5. As a result, the first four quarterly values become 25. Similarly, the proportional annual discrepancies for the second and third year are 10 and 5, which produces quarterly values of 100 and 50.

The pro-rata method produces consistent time series, with large gaps between quarter 4 and quarter 5 and between quarter 8 and 9. The Denton method, however yields much more smooth results. Obviously, the changes between the last quarters of one year and the first quarter of the next year are much better preserved.

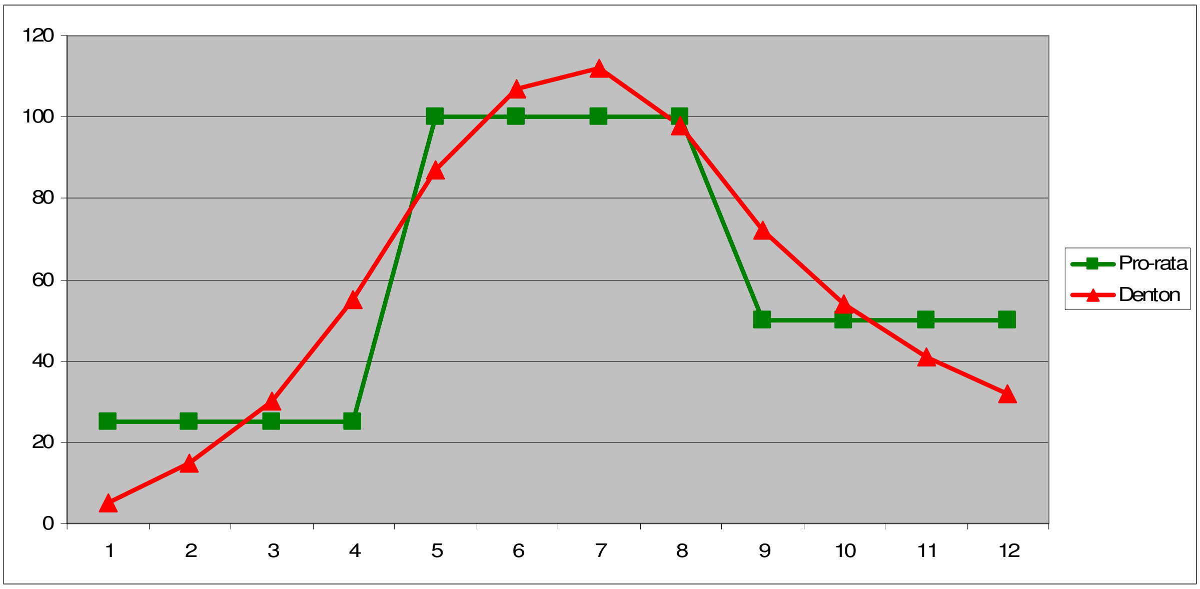

Figure 2. A comparison of pro-rata and Denton.

There are several variants of the Denton method that differ in the way how the changes of the sub-annual data are defined.

The additive first difference variant keeps the additive benchmark corrections as constant as possible. Suppose the benchmark correction of the first quarter is +10. Then, all benchmark corrections will be as close as possible to +10. As a result, when displayed in a graph, the benchmarked time series will be parallel to the initial series.

The proportional fist difference variant keeps the ratio of the benchmarked series to the original series as constant as possible. That means if the benchmark correction of the first quarter is +10%, then all other benchmark corrections will also be as close as possible to +10%. Consequently, the percentual change between two successive sub-annual periods will be preserved as much as possible.

In mathematical terms: when s_t, denotes the original time series, θ_t stands for the benchmarked series and Δ expresses the first-difference operator ($\Delta s_t = s_t - s_{t-1}$):

- the additive variant minimises the sums of squares of $\Delta(\theta_t - s_t)$;

- the proportional variant minimises the sum of squares of $\Delta(\theta_t / s_t)$

and both are subject to annual benchmarks.

The proportional model is often preferred for applications to economic data, especially for non-negative variables (e.g., sales). A reason for this is that this model better preserves the percentual seasonal changes. Another reason is that a proportional model is less likely to produce negative values for non-negative variables. An explanation for this is that an additive model corrects the sub-annual values as equally as possible, regardless of their relative values, while a proportional model corrects higher values more than lower values in an absolute sense (Dagum and Cholette, 2006, page 147).

However an additive model should be used:

- For time series that include zeros. The proportional model is not properly defined for such time series, because the ratio between a benchmarked and an initial value does not exist when an initial value is zero.
- For time series that involve both positive and negative values. A proportional model may yield undesirable results, for instance that all initial values are multiplied by some negative constant, so that all positive values become negative and vice versa.

Initially, the Denton method was proposed for univariate data, Denton (1971). Several extensions are described in the literature, for instance:

- Di Fonzo and Marini (2003) have extended the Denton method for the multivariate case;
- Bikker and Buijtenhek (2006) have added reliability weights to a multivariate Denton method;
- Bikker et al. (2010) have added inequality constraints, soft constraints and ratio constraints to a multivariate Denton model.

In addition to the annual alignment, multivariate Denton methods also allow for *contemporaneous constraints*, i.e., constraints between different variables that should hold at one time period. For instance: for each time period total supply should be equal to total use.

In the extended multivariate Denton method, by Bikker et al. (2010), weights can be used to differentiate the variable by the reliability of its source. As a consequence it can be established that variables that are considered highly reliable can be adjusted less than variables that are considered less reliable.

Another extension is for soft constraints, i.e., constraints that should approximately. These kind of constraints can be used to include subject matter knowledge in the model. For instance: the value of the stocks of some perishable good should be approximately equal to zero, since these kind of goods are usually not kept in stock. A nonzero value, is allowed, but the soft restriction prevents the occurrence of an undesirably high outcome.

Another possibility is to model the annual alignment as a soft constraint, for instance because some annual figure is considered not very reliable. As a consequence the annual data may be adjusted. A benchmarking problem with soft annual alignments is called *nonbinding benchmarking* by Dagum and Cholette (2006). Weights can be attached to soft constraints to determine the relative importance of each constraint.

Furthermore ratio constraints can be included in the model. This is a usual feature since ratio types of interrelationships may play an important role in practical applications. For instance, in the national accounts, it is often assumed that the ratio between production and intermediate use is quite constant over time.

Finally, it is possible to include inequality constraints. A very commonly used type of inequality is the requirement that values cannot be negative.

3. Preparatory phase

4. Examples – not tool specific

4.1 Example: the univariate Denton method

To illustrate the univariate model, we use an artificial data set of twelve quarters and three annual totals. The quarterly figures were chosen to include pronounced changes that follow the seasons. They must then be modified in a way that the sums of the four quarters for each year exactly equal the corresponding annual totals. We imposed no other constraints.

Table 1. Input and Output of a fictitious example

	Input; original data		**Output; benchmarked data**		
	Quarterly data	**Annual totals**	**Denton Proportional**	**Denton Additive**	**Pro rata**
Year 1	50	200	20	-11	25
	100		43	43	50
	150		74	102	75
	100		63	66	50
Year 2	50	500	41	33	63
	100		103	107	125
	150		193	187	188
	100		163	172	125
Year 3	50	1000	101	164	125
	100		237	245	250
	150		392	316	375
	100		271	276	250

The results of a proportional Denton method, an additive Denton method and a pro rata method (after rounding) are shown in table 1. A graphical illustration of these results is given in figure 3.

Due to the higher annual totals, the benchmarked values of the third year are higher than those of the first year. Further, the results show that

- although all original data are positive, the additive Denton method produces a negative value (-11 in quarter 1). This is why a proportional method is preferred for most economic data;
- The Denton methods preserve as much as possible the seasonal pattern, while the pro rata method suffers from the step problem. It does not preserve the original decline between quarter 4 and 5 and quarter 8 and 9;

- In accordance with the theory that the proportional Denton method corrects seasonally higher values more than seasonally lower ones, the benchmarking corrections for each third quarter are the largest for the proportional Denton method, compared to the other two methods. On the other hand, the benchmarking corrections for the additive Denton method are the most constant over time.

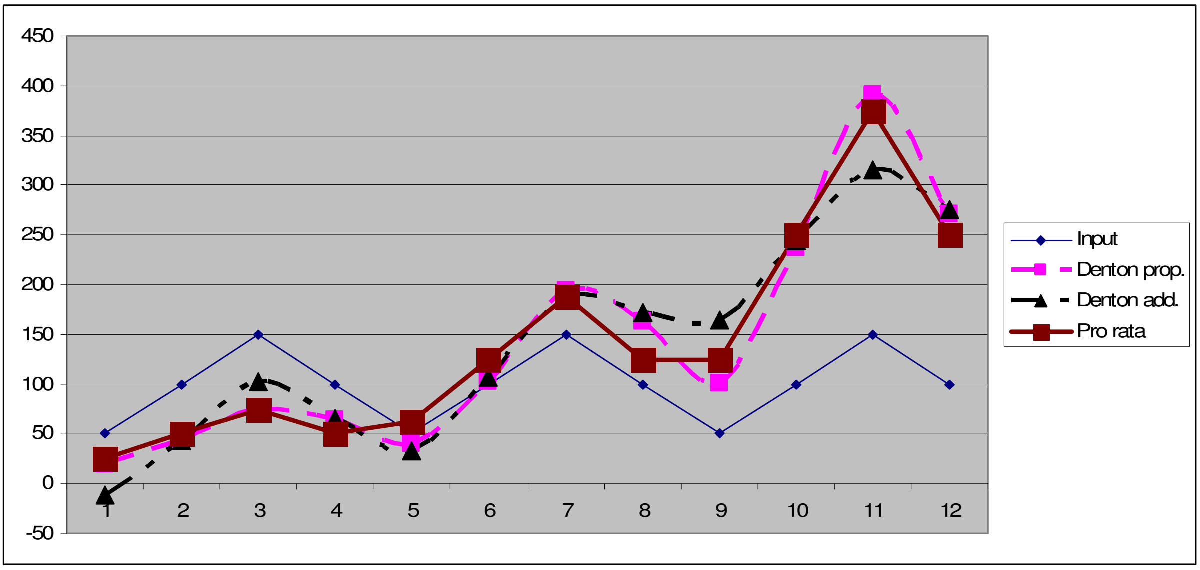

Figure 3. Graphical illustration of the results of table 1.

4.2 Example: the multivariate Denton method

Let us consider a benchmarking problem, consisting of 12 quarters and two time series x_1 and x_2. Suppose initially, each quarterly value is 10 and that annual benchmarks are available for both time series. These are 50, 75 and 95 for the three consecutive years and both time series. Now assume that the first annual alignment is binding, whereas the second and the third are not.

This example is not very realistic, we intentionally choose for large discrepancies between the quarterly and annual data in order to illustrate more vividly how the Denton method works.

Furthermore, there is one soft ratio constraint, defined by

$$\hat{x}_{1t} / \hat{x}_{2t} \approx 1.1 \qquad \text{for } t = 1,\ldots,12.$$

and the proportional model will be used for both time series. Note that the soft, ratio constraint is inconsistent with the annual figures of both time series (both time series have the same annual values, which implies a target value of 1 for the ratio). The relative values of the weights of both model components determine their influence on the model outcome.

The results of the benchmarking method, depicted in figure 4, are two time series, whose values increase gradually over time. This increase is due to the connection to the annual benchmarks. Further note as a result of the ratio from the fifth quarter onwards $\hat{x}_{1t}$ increases more rapidly than $\hat{x}_{2t}$. During the first four quarters, the influence of the ratio constraint is negligible, since the quarters of both time series have to strictly add up to the same annual values. In the second and third year the annual alignment is soft, and therefore the ratio constraint is relatively more important than for the first year.

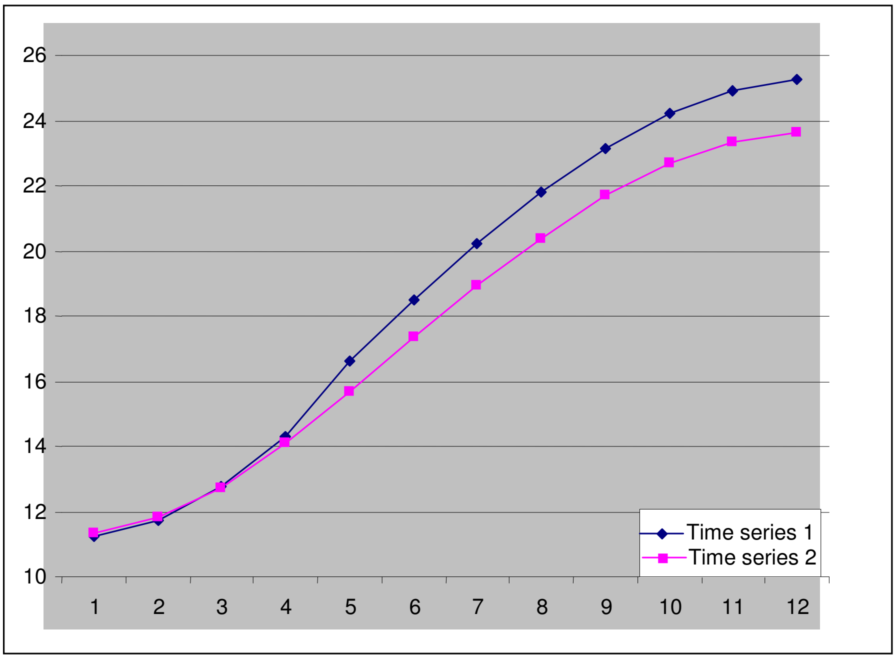

Figure 4. The benchmarked time series.

Table 2 shows that the reconciled annual figures (i.e., the sum of the underlying four quarterly values) of the second and third year closely approximate their target values.

Table 2. Benchmarked annual figures, computed as the sum of four underlying quarterly values;

	Year 1	**Year 2**	**Year 3**
Time Series 1	50.00	77.16	97.61
Time Series 2	50.00	72.32	91.42
Time Series 1 / Time Series 2	1.000	1.067	1.068

5. Examples – tool specific

6. Glossary

For definitions of terms used in this module, please refer to the separate "Glossary" provided as part of the handbook.

7. References

Barcellan, R. and Buono, D. (2002), *ECOTRIM interface user manual*. Eurostat.

Bikker, R. P. and Buijtenhek, S. (2006), Alignment of Quarterly Sector Accounts to Annual Data. Statistics Netherlands, Voorburg. http://www.cbs.nl/NR/rdonlyres/D918B487-45C7-4C3C-ACD0-0E1C86E6CAFA/0/Benchmarking_QSA.pdf

Bikker, R. P., Daalmans, J. A., and Mushkudiani, N. (2010), A multivariate Denton method for benchmarking large data sets. Discussion Paper 10002, Statistics Netherlands, The Hague. http://www.cbs.nl/NR/rdonlyres/7B2387F2-5773-42CF-8C50-5F02B451A2E4/0/201002x10pub.pdf

Bloem, A. M., Dippelsman, R. J., and Maehle, O. N. (2001), *Quarterly National Accounts Manual: Concepts, Data Sources, and Compilation*. International Monetary Fund, Washington, DC. http://www.imf.org/external/pubs/ft/qna/2000/Textbook/index.htm

Boonstra, H. J., De Blois, C. J., and Linders, G. J. (2010), Macro integration with inequality constraints an application to the integration of transport and trade statistics. Statistics Netherlands.

Boot, J. C. G., Feibes, W. and Lisman, J. H. C. (1967), Further methods of derivation of quarterly figures from annual data. *Cahiers Economiques de Bruxelles* **36**, 539–546.

Chen, B. (2007), An Empirical Comparison of Methods for Temporal Distribution and Interpolation at the National Accounts. Bureau of Economic Analysis.

Dagum, E. B. and Cholette, P. A. (2006), *Benchmarking, Temporal Distribution and Reconciliation Methods for Time Series*. Springer, New York.

Denton, F. T. (1971), Adjustment of monthly or quarterly series to annual totals: An Approach based on quadratic minimization. *Journal of the American Statistical Association* **66**, 99–102.

Di Fonzo, T. (1990), The Estimation of *M* Disaggregate Time Series when Contemporaneous and Temporal Aggregates are Known. *The Review of Economics and Statistics* **72**, 178–182.

Di Fonzo, T. and Marini, M. (2003), Benchmarking systems of seasonally adjusted time series according to Denton's movement preservation principle. Working Paper 2003.09, University of Padova. http://www.oecd.org/dataoecd/59/19/21778574.pdf

Fernández, R. B. (1981), Methodological note on the estimation of time series. *Review of Economics and Statistics* **63**, 471–478.

Magnus, J. R., van Tongeren, J. W., and de Vos, A.F. (2000), National Accounts Estimation Using Indicators Analysis. *Review of Income and Wealth* **46**, 329–350.

Magnus, J. R. and Danilov, D. (2008), On the estimation of a large sparse Bayesian system: The Snaer program. *Computational Statistics & Data Analysis* **52**, 4203–4224.

Wei, W. W. S. and Stram, D. O. (1990), Disaggregation of time series models. *Journal of the Royal Statistical Society* **52**, 453–467.

Specific section

8. Purpose of the method

The method is used for Benchmarking which is a specific process step used in the context of macro-integration (cf. "Macro-Integration – Main Module").

9. Recommended use of the method

1. The method can be applied to any problem in which consistency has to be achieved between time series that are published at different frequencies, assuming that the preconditions of section 13 are satisfied.
2. The method may be applied on micro- or macrodata.
3. The method is suitable when the sub-annual series are assumed to be less reliable than the annual time series. The Denton method will revise the sub-annual data and thus, a willingness to revise is necessary.
4. It is especially useful for practical applications in which it is important to preserve the changes of the sub-annual series. For instance: the reconciliation of national accounts data.
5. As mentioned by Bloem et al. (2001) the method is very well suited for large-scale applications.
6. The method should be applied to unbiased source figures. All source figures are consistent with their definitions. They are therefore already adjusted for systematic errors (nonresponse errors, measurement errors, processing errors and conceptual differences). Any errors in the input data (as mentioned in section 12) will propagate to the results.
7. The method may be used in a context of seasonal adjustment, when there are discrepancies between the yearly sums of the raw and the corresponding yearly sums of the seasonally adjusted series. The seasonally adjusted series may be benchmarked to the annual totals derived from the raw series.

10. Possible disadvantages of the method

1. Do not use the Denton method for benchmarking when the annual values are less reliable than the annual sums from the sub-annual series. In this case, using the Denton method will essentially diminish the reliability of sub-annual time series.

11. Variants of the method

1. A Denton method preserves as much as possible the period-to-period changes of the initial sub- annual data. Variants of the Denton method that differ in the way how these changes are defined:

   1.1 The additive model attempts to keep the additive corrections as constant as possible over all periods.

   1.2 The proportional model is designed to preserve the percentage differences as constant as possible over all periods.

1.3 The additive and proportional model may be combined when the Denton method is applied to a multivariate data sets, i.e., the additive method may be applied to some time series, and the proportional model to the other time series.

2. The Denton method can minimise

   2.1 First order differences or

   2.2 Second order differences (differences of differences).

Remark: in the applications that are mentioned in the literature always first-order differences are used.

12. Input data

1. Ds-input1 = a data set (microdata or macrodata) with sub-annual time series of a quantitative variable (Required).
2. Ds-input2 = a data set (microdata or macrodata) with annual totals of the same quantitative variable (Required).

Remark: each time series may comprise one or several annual periods.

13. Logical preconditions

1. Missing values
   1. In Ds-input1 individual missing data values are not allowed.

   Remark: In case of missing source data, one may use a dummy time series, consisting of the value ‘1’ (or any other value) for all sub-annual periods.

   2. In Ds-input2 individual missing data values are allowed (meaning that there is no annual alignment).
2. Erroneous values
   1. All non-empty values can be processed if of the right data type, that is the input must be quantitative values.
3. Other quality related preconditions
   1.
4. Other types of preconditions
   1. One annual period covers a whole number of sub-annual periods.
   2. The annual and sub-annual data describe the same variables.
   3. The constraints (section 14.2) must not be mutually conflicting. In particular this means that: for a multivariate benchmark problem, in which:
      a. all annual alignments (section 14.2.a) are binding;
      b. contemporaneous constraints (section 14.2.b) are defined, that have to be exactly satisfied;

the annual values, that are input to the model (section 12), also have to satisfy all the contemporaneous constraints.

4. The proportional variants of the Denton method (section 11) can only be applied if the initial sub-annual data (section 12.1) do not to contain any zeros.

14. Tuning parameters

1. Weights (Optional). These weights determine which of the time series are adjusted the most (in a multivariate model) and which of the soft constraints are the most stringent. Weights may be omitted; in that case all data and constraints are considered equally reliable.
2. Constraints. These specify the constraints that should be satisfied. The following type of constraints can be used:
   a. Annual alignment: a sum of sub-annual values that should add up to an annual value (Fixed, i.e., this type of constraint is typical for the Denton method).
   b. Contemporaneous constraints (for the multivariate case only): constraints between different time series, within the same period (Optional).

   Technically, a distinction can be made between

   i. soft constraint en hard constrains;
   ii. inequality constraints and equality constraints;
   iii. linear and nonlinear constraints.

15. Recommended use of the individual variants of the method

1. The proportional model (section 11 variant 1.2) is often preferred in application to economic data. The reason for this is that this model better preserves the seasonal changes, as these changes are often measured as a percentage change. However an additive model (section 11 variant 1.1) should be used:
   a. For time series that include zeros, for the proportional model is not defined properly for such time series. The model attempts to preserve the initial percentage changes. A percentage change, however is not defined when the value of the first period is zero.
   b. For time series that involve both positive and negative values. A proportional model may yield undesirable results, for instance that each sub-annual value is multiplied by some negative constant, meaning that all signs change.

16. Output data

1. Ds-output1 = a dataset with reconciled (micro- or macrodata) sub-annual time series.

17. Properties of the output data

1. The reconciled time series (section 16.1) satisfy all restrictions (section 14.2).
2. The seasonal pattern is as close as possible to the seasonal pattern of the initial sub-annual data (section 12.1).

18. Unit of input data suitable for the method

Processing full data sets

19. User interaction - not tool specific

1. Before execution of the method, the tuning parameters and input datasets must be specified.
2. During operation no user interaction is needed, but inspection of quality indicators and subsequent adjustment of tuning parameters and recurrent use is optional.
3. After use of the method the quality indicators and logging should be inspected.

20. Logging indicators

1. The run time of the application.
2. Characteristics of the input data, for instance problem size, the largest discrepancies of the input data towards the constraints.

21. Quality indicators of the output data

1. The most important quality indicator is *how* the sub-annual time series are modified. Of particular interest are the changes made in the first differences, given the starting point of the Denton method. The size of these changes is important, but the trend of the changes in time is particularly important. This trend can usually be assessed fastest by graphical means.

Remark 1: Remind that the Denton method tries to adjust the sub-annual time series in such a way that the adjustments are as smooth as possible over time. If the model still needs to largely adjust the initial changes, this may be an indication that the preconditions of the model are not satisfied and therefore that the method should not have been applied. In other words: the ratio between the reconciled and the raw data should be stable over time.

Remark 2: A quality indicator of an implementation of the Denton method is how accurately the sub-annual series is aligned with the annual series. Numerical error will generally cause these differences to deviate slightly from zero. The differences are not usually a problem as long as they are less than a certain threshold value.

22. Actual use of the method

1. Statistics Netherlands applies the method for reconciliation of quarterly and annual Supply and Use tables, see Bikker et al. (2010).

Interconnections with other modules

23. Themes that refer explicitly to this module

1. Macro-Integration – Main Module

24. Related methods described in other modules

1. Macro-Integration – RAS

2. Macro-Integration – Stone's Method

3. Macro-Integration – Chow-Lin Method for Temporal Disaggregation

Remark: the method of Stone and RAS are also data reconciliation methods, but these are not specially aimed at benchmarking and temporal disaggregation. Many other methods for benchmarking and temporal disaggregation are given in the literature. For an overview we refer to Bloem et al. (2001).

25. Mathematical techniques used by the method described in this module

1. Quadratic optimisation under linear constraints

26. GSBPM phases where the method described in this module is used

1. GSBPM phase 6.2 "Validate Outputs"

27. Tools that implement the method described in this module

1. ECOTRIM

Remark: Freely available from: http://circa.europa.eu/Public/irc/dsis/ecotrim/library However, ECOTRIM is not designed for dealing with thousands of time series, it does not include features like weights, ratios, soft constraints, and the possibility to combine the proportional and additive methods of benchmarking into one model.

2. De Kwartaalmachine (Dutch)

Remark: This software has been developed by Statistics Netherlands and is currently used in the reconciliation of their national accounts. This software is designed for large-scale applications of the multivariate Denton method of Bikker et al. (2010), over 200,000 free variables. The software is not freely available. It makes use of XPRESS, a commercial solver for quadratic programming problems. A license of XPRESS is required to use the Kwartaalmachine.

28. Process step performed by the method

Benchmarking

Administrative section

29. Module code

Macro-Integration-M-Denton

30. Version history

Version	Date	Description of changes	Author	Institute
0.1	31-03-2011	first version	Jacco Daalmans	CBS
0.2	25-01-2012	second version	Jacco Daalmans	CBS
0.3	21-06-2013	third version	Jacco Daalmans	CBS
0.3.1	06-09-2013	preliminary release		
0.3.2	09-09-2013	page numbering adjusted		
1.0	26-03-2014	final version within the Memobust project		

31. Template version and print date

Template version used	1.0 p 4 d.d. 22-11-2012
Print date	26-3-2014 13:26